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**RESPONSE AND DAMPING OF PARALLEL  
CLAMPED-CLAMPED BEAMS JOINED  
BY VISCOELASTIC LINKS**

*by*

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TECHNICAL REPORT AFML-TR-67-221

JULY 1967

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# **RESPONSE AND DAMPING OF PARALLEL CLAMPED-CLAMPED BEAMS JOINED BY VISCOELASTIC LINKS**

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## FOREWORD

This report was prepared by the Strength and Dynamics Branch, Metals and Ceramics Division, under Project No. 7351, "Metallic Materials", by Dr. D. I. G. Jones, of the Metals and Ceramics Division (MAMD) and by Mr. A. D. Nashif and Mr. C. M. Cannon of the University of Dayton under Contract No. AF 33(615)-1506, Task No. 735106. Test equipment purchased using the Director's Fund of the Air Force Materials Laboratory was used in the investigation.

This report covers work performed from July 1965 to February 1967.

The manuscript was released by the authors June 1967 for publication.

This technical report has been reviewed and is approved.

A handwritten signature in black ink, appearing to read "W. J. Trapp", with a stylized flourish at the end.

W. J. TRAPP  
Chief, Strength and Dynamics Branch  
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## ABSTRACT

In this report, an analysis is presented for the response and effective damping of two parallel clamped-clamped beams with a single viscoelastic link joining their centers and with various amounts of internal damping in one beam. It is shown that high damping can still be achieved, even when the fundamental frequency of the beam in question is equal to any natural frequency of the other beam.

The experimental investigation of the same configuration is described and it is shown that the theory accurately describes the phenomena occurring for one particular value of the loss factor of the link, which comprised a ring of one commercially available viscoelastic material.

This abstract is subject to special export controls and each transmittal to foreign governments or foreign nationals may be made only with prior approval of the Metals and Ceramics Division, MAM, Air Force Materials Laboratory, Wright-Patterson Air Force Base, Ohio 45433.

# TABLE OF CONTENTS

	Page
I. INTRODUCTION . . . . .	1
II. THEORY OF PARALLEL CLAMPED-CLAMPED BEAMS WITH VISCOELASTIC LINK JOINING THE CENTERS . . . .	2
III. METHOD OF SOLUTION . . . . .	4
IV. EXPERIMENTAL INVESTIGATION	
(i) $\eta_2 = 0$ . . . . .	6
(ii) $\eta_2 \neq 0$ . . . . .	7
V. CONCLUSIONS . . . . .	8
VI. REFERENCES . . . . .	9
TABLES . . . . .	10
ILLUSTRATIONS . . . . .	31

# TABLES

Table		Page
1	Computed Values of the Effective Loss Factor for $\eta_2 = 0$ and $\phi = 0.5$ . . . . .	10
2	Computed Values of the Effective Loss* Factor for $\eta_2 = 0$ and $\phi = 1.0$ . . . . .	13
3	Computed Values of the Effective Loss Factor for $\eta_2 = 0$ and $\phi = 2.0$ . . . . .	16
4	Computed Values of the Effective Loss Factor for $\eta = 0.8$ and $\phi = 1$ . . . . .	19
5	Experimental Values of the Effective Loss Factor for $\eta_2 = 0$ , $\eta = 0.8$ and $\phi = 0.50$ . . . . .	22
6	Experimental Values of the Effective Loss Factor for $\eta_2 = 0$ , $\eta = 0.8$ and $\phi = 1$ . . . . .	24
7	Experimental Values of the Effective Loss Factor for $\eta_2 = 0$ , $\eta = 0.8$ and $\phi = 2.00$ . . . . .	25
8	Experimental Values of the Effective Loss Factor for $\eta = 0.8$ and $\phi = 1$ . . . . .	27

# ILLUSTRATIONS

Figure		Page
1	Idealized link joining parallel beams . . . . .	31
2	Typical transmissibility spectra for $\lambda/\phi < 1$ . . . . .	32
3	Typical transmissibility spectra for $\lambda/\phi > 1$ . . . . .	33
4	Typical graphs of effective loss factor against link stiffness parameter for $\lambda/\phi < 1$ . . . . .	34
5	Typical graphs of effective loss factor against link stiffness parameter for $\lambda/\phi > 1$ . . . . .	35
6	Graphs of optimum effective loss factor against beam stiffness parameter for $\phi = 0.5$ and $\eta_2 = 0$ . . . . .	36
7	Graphs of optimum effective loss factor against beam stiffness parameter for $\phi = 1.0$ and $\eta_2 = 0$ . . . . .	37
8	Graphs of optimum effective loss factor against beam stiffness parameter for $\phi = 2.0$ and $\eta_2 = 0$ . . . . .	38
9	Graph of optimum effective loss factor against beam stiffness parameter for $\phi = 1.0$ and $\eta_2 = 0.12$ . . . . .	39
10	Graph of optimum effective loss factor against beam stiffness parameter for $\phi = 1.0$ and $\eta_2 = 0.20$ . . . . .	40
11	Graph of optimum effective loss factor against beam stiffness parameter for $\phi = 1.0$ and $\eta_2 = 0.50$ . . . . .	41
12	Sketch of apparatus . . . . .	42
13	Photograph of experimental equipment . . . . .	43
14	Typical measured response spectra for $\lambda/\phi < 1$ . . . . .	44
15	Typical measured response spectra for $\lambda/\phi > 1$ . . . . .	45
16	Graphs of experimentally measured values of effective loss factor against link width for $\lambda/\phi < 1$ . . . . .	46
17	Graphs of experimentally measured values of effective loss factor against link width for $\lambda/\phi > 1$ . . . . .	47

# ILLUSTRATIONS (CON'T)

Figure		Page
18	Comparison of experimental and theoretical values of optimum effective loss factors for $\phi = 0.5$ and $\eta_2 = 0$ . . . . .	48
19	Comparison of experimental and theoretical values of optimum effective loss factors for $\phi = 1.0$ and $\eta_2 = 0$ . . . . .	49
20	Comparison of experimental and theoretical values of optimum effective loss factors for $\phi = 2.0$ and $\eta_2 = 0$ . . . . .	50
21	Comparison of experimental and theoretical values of optimum effective loss factors for $\phi = 1.0$ and $\eta_2 = 0.12$ . . . . .	51
22	Comparison of experimental and theoretical values of optimum effective loss factors for $\phi = 1.0$ and $\eta_2 = 0.20$ . . . . .	52

# LIST OF SYMBOLS

$a_i$	See Equation (9)
$A_i$	Arbitrary constant
$A$	Amplification factor at resonance for beam 1
$b$	Breadth of circular link
$b_i$	See Equation (10)
$B_i$	Arbitrary constant
$c_i$	See Equation (11)
$C_i$	Arbitrary constant
$\text{Cos}$	Circular cosine function
$d$	Separation of beams. Also diameter of circular link used in experimental investigation
$D_i$	Arbitrary constant
$\text{exp}$	Exponential function
$E_i$	Young's Modulus of $i$ th beam ( $\text{Lb/in}^2$ )
$f_i$	Fundamental natural frequency of $i$ th beam (cps)
$i$	Square root of minus one or suffix denoting beam ( $i = 1, 2$ )
$I_i$	Second moment of area of $i$ th beam ( $\text{in}^4$ )
$k$	Real part of stiffness of viscoelastic link ( $\text{Lb/in}$ )
$L$	Length of beams (in)
$\text{Sin}$	Circular Sine function
$\text{Sh}$	Hyperbolic Sine function ( $\text{Sinh}$ )
$t$	Time (sec)
$T_1$	Transmissibility of beam 1

$W_i$	Amplitude of transverse vibration of $i$ th beam relative to clamped ends at any point $x$ (in)
$x$	Station along beams, measured from centers (in)
$X$	Amplitude of vibration of clamped ends (in)
$\Gamma$	$k/E_1 I_1 \lambda_1^3$ - nondimensional link stiffness parameter
$\eta$	Loss factor of viscoelastic link
$\eta_2$	Composite loss factor of beam 2
$\eta_s$	Effective loss factor of beam 1
$\lambda$	$E_2 I_2 / E_1 I_1$ - nondimensional parameter defining relative stiffnesses of beams
$\lambda_1, \lambda_2$	See Equations (4) and (5)
$\mu_i$	Mass per unit length of $i$ th beam (Lb/in)
$\tau$	Thickness of link (ring)
$\xi_i$	$\lambda_i L/2$ - nondimensional frequency parameter
$\phi$	$\mu_2/\mu_1$ - nondimensional mass parameter
$\omega$	Circular frequency (rad/sec)

## I. INTRODUCTION

Many complex structures exhibiting vibrational problems contain within them elements which are parallel or substantially so. Examples are (i) commercial aircraft fuselages, where the cabin wall in the passenger area lies essentially parallel to the outer skin, separated from it by the frames and (ii) thin control surfaces, such as rudders or stabilizers, of which the skins are separated by webs and spars.

In such situations, the vibrational characteristics of the parallel elements are often different and, under certain circumstances, the use of viscoelastic links between antinodal parts of parallel elements can, with proper use, lead to the introduction of significant amounts of damping into the structure at the expense of very little added weight.

In order to gain some insight into the possibilities of such a technique, some investigations of the response of parallel beams, both damped and undamped, of different flexural rigidities and weights, joined by such links, have been carried out at the Air Force Materials Laboratory. In the preliminary investigation attention has been concentrated on a pair of parallel clamped-clamped beams with a viscoelastic link joining the centers and with one of the beams damped by a surface treatment, in order to assess the effect of such an additional feature. See [1,2].

A theory of the response of the system is developed and it is shown that substantial amounts of damping can be introduced into the beams by proper choice of the link stiffness, even when a natural frequency of one beam is identical to (or close to) any natural frequency of the other beam, in which case damping can be still achieved if an additional layered treatment is used on one beam only.

An experimental investigation is described in which the theory is verified for links made of one particular, commercially available, viscoelastic material.

## II. THEORY OF PARALLEL CLAMPED-CLAMPED BEAMS WITH

### VISCOELASTIC LINK JOINING THE CENTERS

Consider two parallel clamped-clamped beams joined by a viscoelastic link as shown in Figure 1. If the amplitude of the harmonic vibration of one beam relative to the clamped ends is  $W_1(x)$  and that of the other beam is  $W_2(x)$ , and the supports are vibrating with harmonic displacement  $X \exp(i\omega t)$ , then the equations of motion of the two beams are:

$$E_1 I_1 (d^4 W_1 / dx_1^4) - \mu_1 \omega^2 W_1 = \mu_1 \omega^2 X \quad (1)$$

$$E_2 I_2 (1 + i\eta_2) (d^4 W_2 / dx_2^4) - \mu_2 \omega^2 W_2 = \mu_2 \omega^2 X \quad (2)$$

at all points apart from the points to which the link is attached.

The general solution of Equations (1) and (2) is derived as follows:

$$\begin{aligned} W_i(x) &= A_i \operatorname{Ch}(\lambda_i x) + B_i \operatorname{Sh}(\lambda_i x) + C_i \operatorname{Cos}(\lambda_i x) \\ &+ D_i \operatorname{Sin}(\lambda_i x) - X \quad [i=1,2] \end{aligned} \quad (3)$$

$$\text{where} \quad \lambda_1^4 = \mu_1 \omega^2 / E_1 I_1 \quad (4)$$

$$\text{and} \quad \lambda_2^4 = \mu_2 \omega^2 / E_2 I_2 (1 + i\eta_2) \quad (5)$$

The eight constants  $A_i$ ,  $B_i$ ,  $C_i$  and  $D_i$  are determined from the various boundary conditions. These are that

$$W_i = dW_i/dx = 0 \text{ at } x = \pm L/2, \quad dW_i/dx = 0 \text{ at } x = 0 \text{ and:}$$

$$2E_1 I_1 (d^3 W_1 / dx^3) = k(1 + i\eta)(W_2 - W_1) \quad (6)$$

$$2E_2 (1 + i\eta_2) I_2 (d^3 W_2 / dx^3) = k(1 + i\eta)(W_1 - W_2) \quad (7)$$

at  $x = 0$ . From these conditions, the eight equations for the eight unknown constants are readily set up and solved. After some simplification, the solution  $W_1(x)$  is readily shown to be:

$$\frac{W_1(x)}{X} = \frac{r(1+i\eta) [b_1 c_2 + (b_2 c_1 / \lambda) (\phi / \lambda)^{-3/4}] + 4c_1 a_2}{r(1+i\eta) [a_2 b_1 + (a_1 b_2 / \lambda) (\phi / \lambda)^{-3/4}] + 4a_1 a_2} \quad (8)$$

where

$$a_i = \text{Sh } \xi_i \cos \xi_i + \text{Ch } \xi_i \sin \xi_i \quad (9)$$

$$\begin{aligned} b_i &= a_i [\text{Sh}(2\xi_i x/L) - \sin(2\xi_i x/L)] \\ &\quad - \text{Ch}(2\xi_i x/L) [\text{Ch } \xi_i \cos \xi_i + \text{Sh } \xi_i \sin \xi_i - 1] \\ &\quad - \cos(2\xi_i x/L) [\text{Ch } \xi_i \cos \xi_i - \text{Sh } \xi_i \sin \xi_i - 1] \end{aligned} \quad (10)$$

$$\begin{aligned} c_i &= \sin \xi_i \text{Ch}(2 \xi_i x/L) + \text{Sh } \xi_i \cos(2 \xi_i x/L) - a_i \\ &\quad [i = 1, 2] \end{aligned} \quad (11)$$

where

$$\xi_i = \lambda_i L/2 \quad (12)$$

The transmissibility  $T_1$  is defined as the ratio of the response at any point of beam 1, relative to a fixed point in space, to the input amplitude  $X$ , i.e.

$$T_1 = |W_1 + X| / X \quad (13)$$

### III. METHOD OF SOLUTION

For given values of the ratio  $\phi$  of the beam masses per unit length, beam 1 being taken as reference, the ratio  $\lambda$  of the flexural rigidities, and the beam loss factor  $\eta_2$  the transmissibility  $T_1$  can be expressed as a function of the frequency parameter  $\xi_1$ , the link loss factor  $\eta$  and the link stiffness parameter  $\Gamma$ . (See Equation 13)

The calculations were performed for  $\phi = 0.5, 1.0$  and  $2.0$  for  $\eta_2 = 0$  and  $\phi = 1$  for  $\eta_2 = 0.12, 0.20$  and  $0.5$  and a range of values of  $\lambda$  between  $0.01$  and  $100$  at  $x = 0$ . Transmissibility spectra such as those illustrated in Figures 2 and 3 were obtained by means of a digital computer. The characteristics of the response were found to depend on whether  $\lambda/\phi = 1$ ,  $\lambda/\phi < 1$  or  $\lambda/\phi > 1$ .

If  $\lambda/\phi = 1$ , the first resonant frequency of beam 2 is equal to that of beam 1 and the two beams will always vibrate nearly in phase with each other (exactly for  $\eta_2 = 0$ ). In this case, little deformation occurs in the viscoelastic link and hence little damping can be introduced into the system by means of viscoelastic links. However some damping can be achieved for  $\eta_2 \neq 0$ .

If  $\lambda/\phi < 1$ , the first resonant frequency of beam 2 is always lower than that of beam 1 and the spectra shown in Figure 2 are typical. Figure 2 shows that the amplitude of the low frequency resonance peak is smaller than that of the higher frequency peak for small values of  $\Gamma$  and, as  $\Gamma$  increases, the amplitude of the second peak eventually becomes smaller than that of the first peak. The effective loss factor  $\eta_s$  of a clamped-clamped beam under shaker excitation, for which the clamped ends are vibrated to give the excitation, has been shown [3] to be:

$$\eta_s = 1.32 (A^2 - 1)^{-1/2} \quad (14)$$

where  $A$  is the amplification factor measured at the center of beam 1 at the resonant peaks corresponding to the first mode. Calculated values of  $\eta_s$  for various values of  $\lambda, \phi, \eta, \eta_2$  and  $\Gamma$  are listed in Tables 1-4. Typical graphs illustrating the variation of the effective loss factors of the two peaks with the stiffness parameter  $\Gamma$  are shown in Figure 4 for several values of the link loss factor  $\eta$ . It is seen that, for each  $\eta$ ,

a value of  $\Gamma$  exists for which both peaks will have the same effective loss factor. This loss factor corresponds to the case where the system is "properly tuned" for beam 1, since it represents the maximum loss factor obtainable for given values of  $\lambda$ ,  $\phi$  and  $\eta$ , in the frequency range of the fundamental mode of beam 1. This procedure was followed for various values of  $\lambda/\phi$  between 1 and 0.035. At  $\lambda/\phi = 0.035$ , the first natural frequency of beam 1 is identical to the third natural frequency of beam 2 and the effective loss factor is again small or zero. For values of  $\lambda/\phi < 0.035$ , analysis of the response spectra followed the procedure adopted for values of  $\lambda/\phi$  between 1 and 0.035. However, in this case, the natural frequency of the fundamental mode of beam 1 is higher than the natural frequencies of the first and third modes of beam 2 and hence the predominant peak due to beam 2 was compared instead with the first peak due to beam 1 in order to define the optimum effective loss factor.

On the other hand, when  $\lambda/\phi > 1$ , the first natural frequency of beam 2 is greater than that of beam 1, and the spectra shown in Figure 3 are typical. It is seen that one peak now dominates the response for all values of the stiffness parameter  $\Gamma$ , even though two peaks still exist. Graphs of  $\eta_s$  against  $\Gamma$  for the predominant peak are illustrated in Figure 5.

Depending on whether  $\lambda/\phi < 1$  or  $\lambda/\phi > 1$ , therefore, one may define the point of optimum damping either as that at which the curves of  $\eta_s$  against  $\Gamma$  cross over, or that at which the curve of  $\eta_s$  against  $\Gamma$  has a maximum, respectively. Graphs of the optimum damping so defined were determined in this manner for many values of  $\lambda$ ,  $\phi$  and  $\eta_2$  and the results are plotted in Figures 6 to 11.

#### IV. EXPERIMENTAL INVESTIGATION

(i)  $\eta_2 = 0$

The experiments were carried out for several combinations of parallel clamped-clamped beams joined by viscoelastic links in the center. For these combinations, the weight ratios were  $\phi = 0.5, 1.0$  and  $2.0$  and the range of values of the stiffness ratio was from  $0.02$  to  $50$ . The stiffness ratio  $\lambda [(f_2/f_1)^2]$  was determined by measuring directly the first observable resonant frequencies of the beams with no links. The material used for the viscoelastic links was LD-400, manufactured by the Lord Manufacturing Company, Erie, Pennsylvania, which has a loss factor of approximately  $0.8$  at room temperature and does not vary greatly with temperature in the vicinity of room temperature.

For each set of values of  $\lambda$  and  $\phi$ , two beams were made and clamped in the mounting fixture which was attached to the shaker table as in Figure 12. The circular viscoelastic link, of thickness  $t$ , width  $b$  and outside diameter  $d$ , was attached to both beams to join the centers. An accelerometer was attached to the mounting fixture to measure and control the input acceleration which was kept at a constant amplitude throughout each test. The acceleration output was measured by means of two miniature accelerometers at the center of each beam. An overall view of the experimental apparatus is illustrated in Figure 13. For a given width of the link, the acceleration output was measured and recorded continuously on an X-Y plotter over a wide range of frequencies. The width  $b$  of the link, which is proportional to the stiffness parameter  $\Gamma$ , was then varied in order to obtain the maximum effective loss factor in the same way as described in the Method of Solution. Typical experimental response spectra for beam 1 are illustrated in Figures 14 and 15 for  $\lambda/\phi < 1$  and  $\lambda/\phi > 1$ , respectively. This test procedure was repeated for every value of  $\phi$  and  $\lambda$  used in the tests.

From all the response spectra so obtained, graphs of the effective loss factor of the significant peaks were plotted against the stiffness parameter (here represented by the link width  $b$ ) in the same way as in the reduction of the theoretical results. Measured values of  $\eta_s$  are tabulated in Tables 5-7.

Typical graphs of the effective loss factor measured in the experiments with the link width  $b$  are plotted in Figure 16 and 17 for  $\lambda/\phi < 1$  and  $\lambda/\phi > 1$ , respectively. From graphs such as these, the optimum effective loss factors were determined in the usual manner and plotted against the beam stiffness

parameter  $\lambda$  for  $\phi = 0.5, 1.0$  and  $2.0$  as shown in Figure 18, 19 and 20, for a link loss factor  $\eta$  of  $0.8$ . Theoretically derived curves of  $\eta_s$  versus  $\lambda$  for  $\eta = 0.8$ , taken from Figures 6, 7 and 8 are also shown with the experimental results. It is seen that the agreement between theory and experiment is good.

(ii)  $\eta_2 \neq 0$

The experiments were repeated in the same way as above for several combinations of parallel clamped-clamped beams joined by viscoelastic links at the centers and with damping introduced into beam 2 by treating it with an unconstrained layer of LD-400. The thicknesses of the unconstrained layer and the metal in the treated beams were varied in such a way as to give a  $\phi$  of unity at all times and loss factors in the damped beam of  $0.12$  and  $0.20$ . The range of values of the stiffness ratio  $\lambda$  was from  $0.1$  to  $4$ . The stiffness ratio  $\lambda = (f_2/f_1)^2$  was determined by measuring the first observable resonant frequencies of the beams with no links.

From the experimental response spectra, values of the effective loss factor at the significant peaks for various values of the link width  $b$  were obtained for each value of  $\lambda$ , tabulated in Table 8, and plotted out. The optimum effective loss factor  $\eta_s$  obtained from these curves in the usual way was plotted against  $\lambda$  as illustrated in Figures 21 and 22, with the theoretical curves superimposed for comparison. The experimental results are in good agreement with theory.

## V. CONCLUSIONS

An analysis has been developed for the response of two parallel clamped-clamped beams, with various relative masses and flexural rigidities, connected at the centers by a viscoelastic link. The effect of link stiffness and loss factor, and of the relative masses and flexural rigidities of the beams, on an arbitrarily defined effective loss factor are described. It is shown that there are certain circumstances in which low damping of the system occurs and others in which high damping is introduced. An experimental investigation is described which shows that the analysis accurately predicts the behavior of the system and that high damping can be introduced into a simple structural system by means of viscoelastic links.

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3. Jones, D. I. G., Nashif, A. D. and Adkins, R. L., "Effect of Tuned Dampers on Vibrations of Simple Structures", AIAA J. 5, No. 2, pp 310-315 (1967).

Table 1. Computed Values of the Effective Loss Factor for

$$\eta_2 = 0 \text{ and } \phi = 0.5$$

$\lambda$	$r$	$\eta = 0.2$		$\eta = 0.5$		$\eta = 0.8$		$\eta = 1.0$		$\eta = 2.0$	
		$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$
.010	.1		.019		.045		.070		.084		.130
	.2		.058	.288	.140		.200	.203		.117	
	.3	.153	.117	.183	.288	.162		.140		.080	
	.4	.096	.171	.120	.415	.111		.100		.057	
	.5	.052		.088		.083		.080		.045	
	.6	.051		.062		.067		.064		.038	
.015	.2		.012		.018		.021		.021		.023
	.4		.014		.017		.019		.021		.023
	.6		.017		.019		.021		.021		.023
	.8		.019		.020		.021		.022		.023
	1.0		.020		.021		.022		.022		.023
	1.2		.021		.021		.022		.022		.023
	1.4		.021		.022		.022		.022		.024
	1.6		.022		.022		.022		.023		.024
.020	.2		.011		.018		.025		.030		.035
	.4		.029		.024		.034		.033		.028
	.6		.033		.032		.031		.030		.026
	.8		.031		.030		.029		.028		.025
	1.0		.029		.028		.027		.026		.024
.035	.2		.027		.061		.098		.123		.247
	.4		.049		.118		.183		.224		.335
	.6		.060		.148		.214		.245		.310
	.8		.068		.150		.214		.240		.288
	1.0		.065		.158		.207		.240		.271
	1.2		.080		.141		.207		.220		.245
	1.4		.067		.140		.188		.200		.225
	1.6		.060		.150		.175		.188		.208
.050	.2		.025		.077		.120		.150		.272
	.4	.700	.080		.195		.305		.380	.348	.620
	.6	.429	.128	.620	.215	.548	.479	.465	.576	.245	
	.8	.260	.170	.402	.395	.369	.565	.323	.653	.180	
	1.0	.200	.175	.286	.438	.265	.606	.238	.690	.141	
.070	.2	.795	.036		.088		.135	1.11	.164		.270
	.4	.521	.108	.145	.260		.390		.448	.280	
	.6	.315	.215	.470	.515	.423	.780	.360		.190	1.72
	.8	.188	.325	.298		.278		.250	1.32	.140	
	1.0	.125	.430	.208		.200		.175		.118	

Table 1. (Con't)

$\lambda$	$r$	$\eta = 0.2$		$\eta = 0.5$		$\eta = 0.8$		$\eta = 1.0$		$\eta = 2.0$	
		$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$
.100	.1	.765	.022		.039		.055		.070		.131
	.2	.590	.040	.806	.095		.145		.176		.258
	.3	.455	.080	.666	.185	.680	.275		.319	.268	
	.4	.355	.137	.532	.325	.513	.446	.422		.215	
	.5	.268	.221	.415	.515	.387		.332		.174	
	.6			.325	.760	.300		.261		.143	
	.7			.255		.240		.214		.121	
	.8			.206		.197		.178		.105	
.250	.1	.203	.020	.312	.045	.340	.070	.346	.085	.132	.110
	.2	.143	.065	.220	.145	.235	.200	.225	.218	.122	.230
	.3	.090	.155	.145	.323	.154	.225	.140	.260	.080	.291
	.4	.055	.320	.100		.105	.360	.097	.386	.059	.275
	.5	.040	.570	.072		.080	.510	.072	.449	.045	.232
1.00	.2	.016		.033		.047		.056		.084	
	.4	.014		.034		.050		.057		.062	
	.6	.014		.032		.043		.048		.045	
	.8	.014		.027		.038		.040		.029	
	1.0	.012		.027		.032		.034		.025	
2.5	.2	.018		.035		.070		.090		.174	
	.4	.031		.073		.115		.143		.250	
	.6	.037		.089		.136		.166		.210	
	.8	.041		.096		.144		.170		.175	
	1.0	.042		.096		.140		.163		.175	
	1.2					.135		.150		.152	
5.0	1.4					.124		.137		.132	
	.4	.035		.090		.145		.172		.330	
	.8	.056		.135		.210		.259		.405	
	1.2	.063		.152		.230		.270		.310	
	1.6	.064		.150		.220		.245		.245	
10.0	2.0	.061		.140		.195		.215		.200	
	.4	.038		.095		.150		.188		.367	
	.8	.064		.160		.250		.310		.575	
	1.2	.080		.195		.305		.372		.522	
	1.6	.086		.210		.315		.372		.412	
25	2.0	.087		.207		.302		.340		.335	
	.4	.042		.097		.154		.192		.382	
	.8	.070		.148		.277		.345		.665	
	1.2	.093		.230		.362		.450		.820	
	1.6	.105		.262		.410		.500		.710	
	2.0	.112		.275		.423		.500		.600	

Table 1 (Con't)

$\lambda$	$r$	$\eta = 0.2$		$\eta = 0.5$		$\eta = 0.8$		$\eta = 1.0$		$\eta = 2.0$	
		$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$
50	.4	.040		.099		.157		.196		.390	
	.8	.071		.180		.282		.355		.700	
	1.2	.095		.240		.383		.476		.925	
	1.6	.114		.280		.450		.557		.945	
	2.0	.124		.305		.480		.583		.840	
100	.4	.040		.100		.158		.196		.390	
	.8	.072		.180		.290		.360		.710	
	1.2	.100		.248		.392		.490		.960	
	1.6	.120		.295		.470		.587		1.11	
	2.0	.130		.327		.511		.635		1.05	

Table 2. Computed Values of the Effective Loss Factor for

$$\eta_2 = 0 \text{ and } \psi = 1.0$$

$\lambda$	$\Gamma$	$\eta = 0.2$		$\eta = 0.5$		$\eta = 0.8$		$\eta = 1.0$		$\eta = 2.0$	
		$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$
.010	.2		.024		.057		.092		.114		.230
	.4		.040		.091		.139		.160		.161
	.6		.043		.089		.120		.130		.116
	.8		.038		.078		.098		.107		.093
	1.0		.037		.066		.088		.088		.080
	1.2		.029		.058		.075		.074		.070
.015	.2		.032		.078		.120		.143		.168
	.4		.070	.540	.175	.480	.281	.436	.354	.282	.440
	.6		.095	.405	.214	.368	.326	.330	.360	.204	.325
	.8		.095	.292	.207	.273	.285	.247	.300	.157	.255
	1.0		.089	.212	.190	.207	.240	.195	.248	.121	.210
	1.2		.082	.167	.170	.169	.208	.157	.218	.100	.180
.025	.2	.207	.040	.222	.095	.210	.150	.195	.183	.140	.156
	.4	.200	.110	.145	.272	.133	.440	.132	.830	.080	.975
	.6	.065	.178	.100	.450	.105	.730	.090	.668	.055	.700
	.8	.088	.205	.090	.480	.069	.640	.060	.555	.040	.540
	1.0	.070	.195	.058	.430	.048	.535	.045	.468	.035	.440
	1.2	.046	.180	.040	.272	.035	.460	.035	.403	.030	.320
.035	.2		.012		.013		.014		.015		.017
	.4		.015		.015		.016		.016		.018
	.6		.016		.016		.017		.017		.018
	.8		.017		.017		.017		.017		.018
	1.0		.017		.017		.017		.018		.018
	1.2		.017		.018		.018		.018		.018
.050	.2		.026		.050		.077		.095		.170
	.4		.037		.076		.109		.133		.188
	.6		.041		.088		.122		.130		.160
	.8		.042		.076		.125		.140		.140
	1.0		.034		.083		.131		.135		.122
	1.2		.052		.100		.121		.120		.110
	1.4		.077		.111		.109		.107		.100
	1.6		.100		.110		.098		.098		.093
	1.8		.091		.091		.090		.090		.086
.070	.2		.025		.060		.097		.123		.245
	.4		.055		.130		.207		.260		.472
	.6		.078		.188		.292		.355	.635	.530
	.8		.105		.230		.341		.402	.490	.569
	1.0		.120	.750	.260	.750	.373	.680	.440	.390	.570
	1.2		.133	.600	.276	.600	.402	.549	.450	.320	.576
	1.4		.130	.490	.306	.499	.410	.450	.475	.270	.587
	1.6		.154	.405	.318	.400	.430	.380	.519	.235	.600
	1.8		.183	.340	.318	.350	.470	.329	.523	.207	.620
	2.0		.160	.290	.337	.305	.500	.285	.522	.185	.640

Table 2 (Con't)

$\lambda$	$r$	$\eta = 0.2$		$\eta = 0.5$		$\eta = 0.8$		$\eta = 1.0$		$\eta = 2.0$	
		$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$
.100	.2		.029		.070		.106		.137	1.340	.260
	.4	.710	.070		.165		.262		.325	.565	
	.6	.577	.415		.285		.455		.570	.520	1.020
	.8	.445	.170	.720	.415	.750	.648	.700	.785	.400	1.070
	1.0	.346	.230	.575	.537	.590	.795	.545		.320	1.030
	1.2	.276	.275	.465	.652	.471		.435		.260	.960
	1.4	.215	.350	.376	.770	.383		.355		.220	.860
	1.6	.175	.430	.310		.322		.300		.190	.780
.125	.2	.753	.030		.070		.110		.140	1.160	.270
	.4	.580	.078		.180		.285	.990	.350		
	.6	.462	.137	.738	.335	.800	.530	.780	.650	.460	
	.8	.365	.215	.600	.530	.633	.845	.600	1.050	.350	
	1.0	.280	.307	.480	.748	.498		.460		.270	
	1.2	.224	.417	.386		.397		.370		.220	
	1.4	.180	.560	.313		.325		.300		.180	
.160	.2	.587	.030		.072		.115		.140	.960	.270
	.4	.467	.080	.720	.192	.800	.302	.812	.370		
	.6	.367	.155	.592	.383	.650	.593	.635	.707	.380	
	.8	.277	.265	.472	.655	.510		.480		.290	
	1.0	.215	.405	.375		.395		.372		.230	
	1.2	.170	.595	.300		.315		.295		.180	
.25	.2	.350	.045	.555	.078	.635	.125	.658	.150	.645	.275
	.4	.265	.090	.456	.225	.520	.340	.525	.405	.375	
	.6	.208	.195	.360	.470	.400	.690	.392		.253	
	.8	.150	.360	.280		.305		.292		.185	
	1.0	.120	.605	.220		.238		.225		.145	
	1.2	.091		.175		.190		.185		.120	
.50	.2	.125	.040	.208	.098	.235	.140	.245	.177		
	.4	.070	.135	.135	.313	.155	.255	.157	.320		
	.6	.050	.325	.093	.715	.110	.525	.103	.598		
	.8	.035	.650	.070		.075		.076	.746		
	1.0	.030		.056		.062		.060			
2.0	.2	.018		.037		.057		.070		.125	
	.4	.020		.047		.070		.083		.099	
	.6	.022		.046		.065		.073		.071	
	.8	.019		.042		.056		.061		.055	
	1.0	.020		.037		.048		.051		.046	
	1.2	.015		.033		.042		.045		.039	

Table 2 (Con't)

$\lambda$	$r$	$\eta = 0.2$		$\eta = 0.5$		$\eta = 0.8$		$\eta = 1.0$		$\eta = 2.0$	
		$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$
5.0	.2	.019		.047		.075		.095		.185	
	.4	.033		.082		.130		.162		.302	
	.6	.043		.106		.165		.202		.327	
	.8	.049		.118		.183		.220		.272	
	1.0	.052		.124		.185		.215		.225	
	1.2	.053		.124		.177		.200		.190	
	1.4	.052		.119		.166		.182		.165	
	1.6	.049		.113		.152		.165		.145	
	1.8	.048		.106		.140		.150		.130	
	2.0	.045		.100		.130		.137		.115	
10.0	.4	.038		.092		.145		.181		.355	
	.8	.061		.152		.240		.295		.510	
	1.2	.074		.180		.277		.334		.375	
	1.6	.078		.185		.270		.305		.283	
	2.0	.075		.175		.240		.263		.225	
25	.4	.040		.098		.155		.192		.380	
	.8	.069		.170		.273		.340		.650	
	1.2	.090		.220		.352		.435		.725	
	1.6	.102		.250		.390		.470		.565	
	2.0	.107		.260		.385		.450		.470	
50	.4	.040		.100		.155		.195		.390	
	.8	.070		.178		.283		.352		.690	
	1.2	.095		.235		.378		.470		.900	
	1.6	.112		.278		.437		.540		.845	
	2.0	.120		.297		.463		.555		.730	
100	.4	.042		.100		.157		.199		.390	
	.8	.073		.180		.290		.360		.700	
	1.2	.100		.245		.390		.486		.950	
	1.6	.118		.290		.463		.576		1.060	
	2.0	.130		.320		.503		.620		.975	

Table 3. Computed Values of the Effective Loss Factor for

$$\eta_2 = 0 \text{ and } \psi = 2.0$$

$\lambda$	$r$	$\eta = 0.2$		$\eta = 0.5$		$\eta = 0.8$		$\eta = 1.0$		$\eta = 2.0$	
		$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$
.010	.4		.006		.016		.022		.023		.026
	.8		.006		.016		.024		.025		.026
	1.2		.006		.016		.026		.026		.027
	1.6		.006		.016		.026		.026		.027
	2.0		.006		.016		.026		.026		.027
.017	.2		.019		.048		.078		.096		.160
	.4		.030		.066		.095		.105		.100
	.6		.027		.058		.078		.078		.072
	.8		.022		.052		.059		.061		.060
	1.0		.030		.041		.048		.052		.050
	1.2		.023		.034		.043		.048		.045
.025	.2		.025		.060		.096		.118		.236
	.4		.048		.120		.194		.245		.303
	.6		.064		.183		.238		.262		.215
	.8		.068		.180		.202		.210		.169
	1.0		.062		.135		.169		.170		.138
	1.2		.056		.119		.143		.145		.115
.035	.2		.028		.065		.103		.129		.250
	.4		.057		.145		.171		.294	.420	.535
	.6		.079		.225		.370		.468	.373	.400
	.8		.109		.268		.385		.400	.330	.312
	1.0		.111		.255		.325		.529	.280	.253
	1.2		.105		.225		.275		.280	.239	.215
	1.4		.095		.197		.237		.240	.205	.185
.050	.4		.065	.208	.170	.213	.265	.218	.331	.237	.810
	.8		.143	.210	.374	.235	.615	.245	.670	.235	.495
	1.2		.163	.280	.370	.274	.460	.270	.458	.200	.340
	1.6		.140	.300	.290	.285	.340	.275	.345	.175	.258
	2.0		.110	.295	.235	.283	.270	.270	.272	.160	.209
.060	.4	.110	.070	.110	.180	.110		.105		.090	
	.8	.160	.160	.132	.420	.108		.095		.065	
	1.2	.195	.255	.118	.455	.081		.070		.054	
	1.6	.170	.110	.170	.360	.060		.054		.049	
	2.0	.140	.040	.148	.285	.045		.045		.045	
.080	.4	.090	.090	.090		.097		.100		.125	
	.8	.053	.186	.090		.118		.132		.162	
	1.2	.125	.240	.135		.172		.175		.180	
	1.6		.225	.180		.200		.195		.184	
	2.0		.190	.190		.206		.199		.185	

Table 3 (Con't)

$\lambda$	$r$	$\eta = 0.2$		$\eta = 0.5$		$\eta = 0.8$		$\eta = 1.0$		$\eta = 2.0$	
		$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$
.100	.4		.043		.110		.173		.210		.290
	.8		.092		.120		.300		.295		.270
	1.2		.375		.299		.276		.272		.255
	1.6		.250		.257		.260		.260		.220
	2.0		.200		.240		.250		.253		.195
.120	.4	.807	.053		.130		.205		.258		.445
	.8	.660	.125		.287		.407		.453		.340
	1.2	.525	.290		.460		.440		.400		.247
	1.6	.400	.415		.430		.325		.275		.200
	2.0	.320	.375		.275		.225		.210		.193
.160	.2	.800	.025		.061	1.280	.100		.125		.245
	.4	.700	.060		.145	1.140	.230		.288		.570
	.6	.572	.100		.243	1.060	.390		.480		.710
	.8	.540	.152		.372	.970	.580		.695		.560
	1.0	.450	.227	.770	.533	.880	.730		.748		.452
	1.2	.405	.360	.700	.700	.790	.735		.653		.370
	1.4	.352	.628	.625	.756	.700	.630		.557		.310
	1.6	.320		.560	.650	.625	.535		.455		.265
.200	.2	.795	.025		.065		.100	1.100	.135	1.160	.255
	.4	.525	.108		.150		.240	1.020	.300	1.035	.580
	.6	.515	.170	.785	.265		.427	.930	.535	.870	.920
	.8	.415	.257	.705	.420	.820	.673	.840	.840	.700	.780
	1.0	.385	.395	.650	.650	.740		.750	1.060	.570	.600
	1.2	.320	.685	.580		.663		.660	.950	.470	.480
	1.4	.290		.520		.589		.580	.760	.400	.400
	1.6	.255		.470		.523		.510	.625	.370	.260
.25	.2	.572	.025	.780	.065		.102		.125	.990	.250
	.4	.451	.053	.740	.155	.834	.247		.306	.880	.580
	.6	.400	.114	.650	.282	.755	.450	.787	.560	.725	
	.8	.339	.184	.595	.455	.685	.738	.706	.928	.590	
	1.0	.291	.280	.530	.706	.615		.625		.480	
	1.2	.269	.435	.475		.545		.550		.400	
	1.4	.225	.737	.422		.485		.483		.340	
.35	.2	.490	.027	.608	.068	.685	.104	.715	.130	.760	.260
	.4	.296	.067	.533	.162	.620	.259	.648	.320	.660	.580
	.6	.277	.123	.472	.302	.560	.480	.582	.593	.540	
	.8	.235	.200	.425	.506	.498	.815	.517		.430	
	1.0	.195	.315	.373		.442		.450		.350	
	1.2	.170	.490	.330		.390		.395		.290	
.50	.2	.381	.026	.450	.068	.501	.106	.525	.135	.543	.260
	.4	.188	.070	.355	.175	.426	.275	.450	.338	.455	.572
	.6	.172	.132	.313	.325	.379	.575	.393	.630	.360	
	.8	.155	.225	.275	.565	.325		.340	.282		
	1.0	.130	.358	.235		.283		.290	.230		

Table 3 (Con't)

$\lambda$	$r$	$\eta = 0.2$		$\eta = 0.5$		$\eta = 0.8$		$\eta = 1.0$		$\eta = 2.0$	
		$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$
1.0	.2	.105	.034	.152	.078	.180	.122	.195	.150	.195	.270
	.4	.075	.089	.120	.217	.140	.330	.145	.393	.133	
	.6	.047	.182	.092		.110	.640	.112		.093	
	.8	.035	.327	.070		.090		.088		.071	
	1.0	.036		.062		.070		.075		.060	
5.0	.2	.018		.044		.070		.087		.167	
	.4	.028		.070		.110		.133		.191	
	.6	.034		.080		.120		.140		.132	
	.8	.034		.079		.110		.121		.100	
	1.0	.032		.072		.096		.100		.080	
	1.2	.030		.064		.082		.085		.072	
10.0	.2	.020		.050		.077		.096		.192	
	.4	.036		.089		.137		.172		.330	
	.6	.048		.115		.182		.227		.390	
	.8	.055		.135		.210		.255		.270	
	1.0	.060		.145		.220		.260		.220	
	1.2	.062		.145		.210		.235		.182	
	1.4	.061		.140		.194		.207		.158	
	1.6	.058		.132		.175		.180		.139	
25.0	.4	.040		.095		.150		.190		.375	
	.8	.067		.165		.262		.328		.613	
	1.2	.085		.210		.330		.402		.460	
	1.6	.094		.226		.342		.400		.344	
	2.0	.094		.225		.315		.340		.275	
50.0	.4	.041		.098		.156		.195		.380	
	.8	.070		.175		.280		.346		.670	
	1.2	.095		.230		.358		.455		.820	
	1.6	.107		.265		.419		.509		.660	
	2.0	.115		.280		.427		.498		.540	
	2.4	.115		.277		.407		.455		.460	
	2.8	.114		.250		.375		.415		.405	
100.0	.4	.044		.100		.158		.188		.390	
	.8	.072		.180		.286		.357		.700	
	1.2	.097		.241		.385		.480		.930	
	1.6	.115		.285		.452		.563		.950	
	2.0	.126		.310		.487		.593		.850	
	2.4	.131		.320		.488		.580		.760	
	2.8	.131		.317		.475		.553		.700	
	3.2	.130		.310		.453		.523		.640	
	3.6	.128		.300		.432		.495		.600	
	4.0	.123		.288		.414		.472		.550	

Table 4. Computed Values of the Effective Loss Factor

for  $n = 0.8$ ,  $\phi = 1$ 

$\lambda$	$r$	$\eta_2 = 0.12$		$\eta_2 = 0.20$		$\eta_2 = 0.50$	
		$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$
0.01	0.2	-	0.095	-	0.100	-	0.110
	0.4	-	0.175	-	0.195	-	0.235
	0.6	-	0.210	-	0.260	-	0.355
	0.8	0.540	0.230	0.560	0.305	0.525	0.460
	1.0	0.460	0.245	0.495	0.335	0.490	0.545
	1.2	0.395	0.255	0.435	0.360	0.470	-
	1.4	0.350	0.260	0.395	0.375	0.440	-
	1.6	0.310	0.270	0.355	0.390	0.420	-
	1.8	0.280	0.270	0.330	0.405	0.400	-
	2.0	0.255	0.275	0.310	0.410	0.395	-
0.015	0.2	-	0.120	-	0.120	-	0.120
	0.4	-	0.290	-	0.290	-	0.280
	0.6	0.450	0.415	0.465	0.455	-	0.425
	0.8	0.380	0.450	0.425	0.545	-	0.455
	1.0	0.320	0.460	0.375	0.590	-	0.452
	1.2	0.280	0.465	0.335	0.625	-	0.435
	1.4	-	-	-	-	-	-
0.020	0.2	-	0.130	-	0.130	-	0.125
	0.4	0.363	0.355	-	0.340	-	0.290
	0.6	0.335	0.620	0.650	0.370	-	0.395
	0.8	0.290	0.665	0.790	0.350	-	0.430
	1.0	0.250	0.665	0.850	0.320	-	0.445
	1.2	-	-	-	0.300	-	0.445
	1.4	-	-	-	0.285	-	0.440
	1.6	-	-	-	-	-	-
0.025	0.2	0.210	0.150	-	-	-	-
	0.4	0.265	0.540	-	0.285	-	0.270
	0.6	0.255	0.820	-	0.310	-	0.370
	0.8	-	-	-	0.310	-	0.430
	1.0	-	-	-	0.310	-	0.465
	1.2	-	-	-	0.300	-	0.485
	1.4	-	-	-	0.295	-	0.495
	1.6	-	-	-	0.290	-	0.500
	1.8	-	-	-	0.285	-	0.502
	2.0	-	-	-	0.280	-	0.505
0.030	0.4	-	0.200	-	0.235	-	0.255
	0.8	-	0.225	-	0.305	-	0.440
	1.2	-	0.225	-	0.325	-	0.540
	1.6	-	0.222	-	0.335	-	0.585
	2.0	-	0.200	-	0.338	-	0.610

Table 4. (Con't)

$\lambda$	$r$	$\eta_2 = 0.12$		$\eta_2 = 0.20$		$\eta_2 = 0.50$	
		$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$
0.040	1.2	-	0.300	-	0.425	-	0.675
	1.4	-	0.315	-	0.460	0.885	0.750
	1.6	-	0.330	-	0.490	0.795	0.805
	1.8	-	0.340	-	0.505	0.730	0.850
	2.0	-	0.355	-	0.525	0.675	0.890
0.05	1.0	-	0.380	-	0.500	-	0.700
	1.2	0.800	0.430	0.840	0.580	0.890	0.839
	1.4	0.675	0.475	0.715	0.650	0.785	1.000
	1.6	0.585	0.525	0.625	0.720	0.700	-
	1.8	0.510	0.550	0.555	0.780	0.640	-
	2.0	0.455	0.585	0.500	0.830	0.590	-
0.07	0.8	-	0.500	-	0.565	-	0.655
	1.0	0.810	0.645	0.850	0.755	0.870	0.960
	1.2	0.665	0.795	0.670	0.960	0.755	-
	1.4	0.555	-	0.595	-	0.660	-
	1.6	0.475	-	0.515	-	0.590	-
0.10	0.4	-	0.265	-	0.270	-	0.275
	0.6	-	0.480	-	0.480	-	0.490
	0.8	0.825	0.730	0.850	0.760	0.860	0.750
	1.0	0.655	-	0.690	-	0.725	-
	1.2	0.535	-	0.570	-	0.625	-
0.15	0.2	-	0.115	-	0.115	-	0.115
	0.4	-	0.295	-	0.295	-	0.290
	0.6	0.785	0.570	0.820	0.560	0.780	0.525
	0.8	0.615	-	0.650	0.935	0.665	0.700
	1.0	0.490	-	0.525	-	0.575	-
0.25	0.2	0.850	0.120	0.910	0.120	-	0.120
	0.4	0.645	0.330	0.690	0.320	-	0.300
	0.6	0.500	0.640	0.535	0.610	0.500	-
	0.8	-	-	0.420	-	0.470	-
	1.0	-	-	0.345	-	0.440	-
0.50	0.2	0.375	0.140	-	0.140	-	0.115
	0.4	0.255	0.370	0.295	0.400	0.245	0.360
	0.6	0.190	-	0.225	-	0.275	-
	0.8	0.150	-	0.190	-	0.270	-
1.00	0.2	-	0.090	-	0.058	-	0.075
	0.6	-	0.110	-	0.082	-	0.150
	1.0	-	0.115	-	0.090	-	0.180
	1.4	-	0.116	-	0.093	-	0.195
	1.8	-	0.117	-	0.094	-	0.202

Table 4 (Con't)

$\lambda$	$r$	$\eta_2 = 0.12$		$\eta_2 = 0.20$		$\eta_2 = 0.50$	
		$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$	$\eta_{s1}$	$\eta_{s2}$
1.50	0.4	-	0.056	-	0.075	-	0.105
	0.8	-	0.061	-	0.085	-	0.155
	1.2	-	0.062	-	0.095	-	0.180
	1.6	-	0.063	-	0.097	-	0.200
	2.0	-	0.064	-	0.100	-	0.215
2.50	0.4	-	0.100	-	0.105	-	0.120
	0.8	-	0.110	-	0.150	-	0.170
	1.2	-	0.105	-	0.155	-	0.200
	1.6	-	0.100	-	0.155	-	0.220
	2.0	-	0.095	-	0.153	-	0.235
5.0	0.4	-	0.130	-	0.132	-	0.140
	0.8	-	0.190	-	0.195	-	0.217
	1.2	-	0.195	-	0.210	-	0.257
	1.6	-	-	-	0.205	-	0.277
	2.0	-	-	-	0.197	-	0.285
10	0.4	-	0.145	-	0.150	-	0.150
	0.8	-	0.240	-	0.240	-	0.250
	1.2	-	0.285	-	0.290	-	0.310
	1.6	-	0.285	-	0.300	-	0.345
	2.0	-	0.265	-	0.290	-	0.355
25	0.4	-	0.155	-	0.155	-	0.155
	0.8	-	0.272	-	0.275	-	0.275
	1.2	-	0.355	-	0.355	-	0.365
	1.6	-	0.395	-	0.400	-	0.420
	2.0	-	0.400	-	0.410	-	0.440
50	0.4	-	0.160	-	0.155	-	0.160
	0.8	-	0.280	-	0.285	-	0.285
	1.2	-	0.380	-	0.380	-	0.385
	1.6	-	0.440	-	0.440	-	0.450
	2.0	-	0.465	-	0.475	-	0.490
100	0.4	-	0.155	-	0.160	-	0.160
	0.8	-	0.290	-	0.290	-	0.290
	1.2	-	0.390	-	0.390	-	0.395
	1.6	-	0.465	-	0.465	-	0.470
	2.0	-	0.505	-	0.510	-	0.515

Table 5. Experimental Values of the Effective Loss

Factor for  $\eta_2 = 0$ ,  $\eta = 0.8$ ,  $\phi = 0.50$ 

$\lambda$	$\tau$	$b$	$\eta_{s1}$	$\eta_{s2}$	$\lambda$	$\tau$	$b$	$\eta_{s1}$	$\eta_{s2}$
.0194	.125	0.25	.056	-	0.222	.125	0.125	-	.025
		.50	.054	-			.250	.160	.287
		.75	.053	-			.50	.065	-
		1.00	.062	-			1.00	.045	-
		1.25	.075	-					
		1.50	.079	-					
.0194	.125	0.25	.110	-	1.125	.078	0.125	.039	-
		.50	.043	-			.25	.047	-
		.75	.050	-			.50	.046	-
		1.00	.043	-			.75	.043	-
		1.25	.047	-			1.00	.034	-
.0353	.078	0.25	.815	.105	2.00	.078	0.125	.030	-
		.50	.615	.175			.25	.045	-
		.75	.340	.235			.50	.050	-
		1.00	.265	.305			1.00	.045	-
		1.25	.230	.330			1.25	.048	-
.0353	.125	0.25	-	.087	4.5	.078	0.125	.082	-
		.50	.762	.232			.25	.107	-
		.75	.673	.275			.50	.140	-
		1.00	.555	.296			.75	.122	-
		1.25	.505	.332			1.00	.100	-
.0555	.078	0.50	-	.085	7.1	.078	0.125	.140	-
		1.00	1.130	.140			.25	.155	-
		1.50	.780	.265			.50	.195	-
		2.00	.438	.535			.75	.202	-
							1.00	.170	-
							1.25	.145	-
.0555	.125	0.75	.380	.510	.222	.078	0.75	.550	.060
		1.00	.380	.575			1.00	.360	.215
		1.25	.205	1.18			1.25	.370	.187
		1.50	.260	1.06			1.50	.270	.675
.125	.078	0.125	.507	.110	.125	.078	0.125	.507	.110
		.25	.475	.185			.25	.475	.185
		.50	.360	.475			.50	.360	.475
		.75	.410	.360			.75	.410	.360
		1.25	.270	1.06			1.25	.270	1.06
		1.50	.180	-			1.50	.180	-
.125	.078	0.125	.470	.110	.125	.078	0.125	.470	.110
		.25	-	.227			.25	-	.227
		.50	.262	.620			.50	.262	.620
		1.00	.152	-			1.00	.152	-
		1.50	.092	-			1.50	.092	-

Table 5 (Con't)

$\lambda$	$\tau$	$b$	$\eta_{s1}$	$\eta_{s2}$	$\lambda$	$\tau$	$b$	$\eta_{s1}$	$\eta_{s2}$
12.9	.078	0.50	.290	-	12.9	.078	0.50	.240	-
		.75	.305	-			.75	.260	-
		1.00	.270	-			1.00	.280	-
		1.25	.310	-			1.25	.280	-
		1.50	.228	-			1.50	.270	-
		2.00	.200	-			2.00	.235	-

Table 6. Experimental Values of the Effective Loss Factor

$$\eta_2 = 0, \eta = 0.8, \phi = 1$$

$\lambda$	$\tau$	$b$	$\eta_{s1}$	$\eta_{s2}$
0.0625	0.125	0.21	.388	-
0.104	0.035	0.50	.428	.428
0.104	0.062	0.315	.475	.475
0.104				
0.25	0.062	0.415	.375	.375
0.25	0.062	0.22	.424	.424
0.25				
0.56	0.020	0.48	.227	-
1.00	.028	0.26	.013	.013
1.8	.020	0.38	.086	-
4.00	.062	0.29	.279	-
4.00	.062	0.25	.21	-
4.5				
9.6				
9.61	.035	0.55	.26	-
9.61	.062	0.13	.319	-
16.0	.078	0.40	.337	-
39.0	.078	0.15	.370	-

Table 7. Experimental Values of the Effective Loss Factor

$$\eta_2 = 0, \eta = 0.8, \phi = 2.00$$

$\lambda$	$\tau$	b	$\eta_{s1}$	$\eta_{s2}$	$\lambda$	$\tau$	b	$\eta_{s1}$	$\eta_{s2}$
0.141	.078	0.125	.675	.070	4.5	.078	0.25	.072	-
		.25	.675	.125			.50	.105	-
		.50	.605	.250			.75	.105	-
		.75	.575	.270			1.00	.120	-
		1.00	.505	.385			1.25	.125	-
		1.50	.260	1.06			1.50	.085	-
0.22	.078	0.25	.550	.143	4.5	.125	0.125	.055	-
		.50	.467	.270			.25	.075	-
		.75	.505	.307			.50	.045	-
		1.00	.330	.602			1.00	.032	-
		1.25	.405	.440	8.0	.078	0.125	.115	-
		1.50	.260	1.180			.25	.137	-
0.22	.078	0.125	-	.055			.50	.135	-
		.25	.960	.102			1.00	.100	-
		.50	.762	.250			1.25	.090	-
		.75	.720	.320	8.0	.078	0.125	.125	-
		1.00	.550	.550			.25	.147	-
		1.25	.605	.450			.50	.157	-
		1.50	.485	.106			.75	.157	-
0.50	.078	0.25	.160	.780			1.25	.140	-
		.50	.180	1.99			1.50	.110	-
		.75	.160	1.46	18.0	.078	0.50	.300	-
		1.00	.130	-			1.00	.300	-
		1.25	.120	-			1.50	.285	-
		1.50	.095	-			2.00	.210	-
0.50	.078	0.25	.468	.145	18.0	.078	1.00	.287	-
		.50	.332	.375			1.25	.295	-
		.75	.245	.605			1.50	.293	-
		1.00	.151	-			2.00	.260	-
0.89	.078	0.125	.190	.063	28.4	.078	0.125	.096	-
		.25	.170	.140			.25	.164	-
		.50	.125	.320			.50	.325	-
		1.00	.072	-			.75	.420	-
		1.25	.080	-			1.00	.380	-
0.89	.078	0.125	.137	.084			1.25	.375	-
		.25	.106	.178			1.50	.405	-
		.50	.078	.360	28.4	.125	0.25	.210	-
		.75	.070	.380			.50	.255	-
		1.00	.047	-					

Table 7. (Con't)

$\lambda$	$\tau$	b	$\eta_{s1}$	$\eta_{s2}$	$\lambda$	$\tau$	b	$\eta_{s1}$	$\eta_{s2}$
28.4	.125	.75	.280	-	51.7	.078	0.25	.178	-
		1.00	.320	-			.50	.240	-
		1.25	.310	-			.75	.315	-
		1.50	.255	-			1.00	.330	-
51.7	.078	0.25	.300	-			1.25	.345	-
		.50	.390	-					
		.75	.400	-					
		1.00	.425	-					
		1.25	.480	-					
		1.50	.380	-					

Table 8. Experimental Values of the Effective Loss Factor  
for  $\eta = 0.8$ ,  $\phi = 1$

$\lambda$	$\tau$	$b$	$\eta_2 = 0.12$ $\eta_{s1}$	$\eta_{s2}$	$\lambda$	$\tau$	$b$	$\eta_2 = 0.12$ $\eta_{s1}$	$\eta_{s2}$
0.207	.020	0.125	.023	.017	0.634	.078	0.25	.405	.035
		.25	.023	.024			.50	.340	.065
		.50	.029	.041			.75	.260	.157
		.75	.024	.060			1.00	.240	.230
							1.25	.225	.295
0.272	.078	0.125	.605	.067	0.653	.020	0.125	.201	.011
		.25	.465	.190			.25	.196	.012
		.50	.340	.435			.50	.185	.024
		.75	.340	.405			.75	.180	.037
0.305	.078	0.125	.435	.086			1.00	.175	.051
		.25	.390	.185			1.25	.164	.067
		.50	.280	.294			1.50	.158	.082
		.75	.250	.406			2.00	.123	.175
		1.00	.155	-					
0.305	.035	1.25	.405	.110	0.653	.020	0.25	.259	.019
		1.50	.350	.140			.50	.249	.039
		1.75	.340	.175			.75	.223	.051
		2.00	.337	.250			1.00	.201	.073
							1.25	.201	.079
0.602	.035	0.25	.310	.050			1.50	.191	.114
		.50	.260	.125			2.00	.156	.227
		.75	.220	.210	0.653	.035	0.125	.210	.025
		1.00	.185	.340			.25	.196	.041
		1.25	.165	.435			.50	.130	.120
0.632	.020	0.125	.136	.036			.75	.134	.162
		.25	.130	.055			1.00	.100	.308
		.50	.123	.081			1.25	.093	.382
		.75	.117	.105			1.50	.083	.550
		1.00	.113	.143	0.653	.035	0.250	.209	.055
		1.25	.103	.175			.50	.170	.123
		1.50	.097	.240			.75	.151	.185
0.632	.035	0.125	.105	.077			1.00	.128	-
		.25	.102	.124	0.776	.020	0.25	.047	-
		.50	.085	.280			.50	.047	-
		.75	.078	.325			.75	.047	-
0.634	.020	0.125	.100	.067			1.00	.049	-
		.25	.098	.087			1.25	.051	-
		.50	.094	.118			1.50	.047	-
		.75	.091	.151			2.00	.047	-
		1.00	.085	.196					-

Table 8. (Con't)

$\lambda$	$\tau$	$b$	$\eta_2 = 0.12$		$\lambda$	$\tau$	$b$	$\eta_2 = 0.12$	
			$\eta_{s1}$	$\eta_{s2}$				$\eta_{s1}$	$\eta_{s2}$
0.780	.035	0.125	.127	.041	1.62	.078	0.25	.054	-
		.25	.123	.067			.50	.057	-
		.50	.109	.119			.75	.056	-
		.75	.097	.173			1.00	.069	-
0.789	.078	0.25	.270	.060	1.66	.125	1.25	.070	-
		.50	.215	.145			1.50	.068	-
		.75	.208	.160			2.00	.067	-
		1.00	.147	.380			0.125	.058	-
		1.25	.170	.295			.25	.060	-
0.949	.078	1.50	.148	.505	1.94	.078	.50	.062	-
		0.125	.121	-			.75	.063	-
		.25	.124	-			1.00	.067	-
		.50	.119	-			1.25	.061	-
		.75	.106	-			1.50	.060	-
0.95	.078	1.00	.097	-	1.98	.078	0.125	.141	-
		0.25	.071	-			.25	.155	-
		.50	.072	-			.50	.148	-
		.75	.073	-			.75	.135	-
		1.00	.072	-			1.00	.114	-
1.08	.078	1.25	.070	-	2.04	.125	1.25	.112	-
		1.50	.069	-			0.25	.069	-
		0.25	.047	-			.50	.069	-
		.50	.049	-			.75	.068	-
		.75	.054	-			1.00	.063	-
1.08	.125	1.00	.054	-	2.43	.078	1.25	.067	-
		1.25	.053	-			.25	.053	-
		1.50	.053	-			.50	.061	-
		2.00	.054	-			.75	.067	-
		0.25	.049	-			1.00	.074	-
1.44	.125	.50	.050	-	2.43	.078	1.25	.070	-
		.75	.051	-			1.50	.069	-
		1.00	.050	-			2.00	.067	-
		1.25	.050	-			0.25	.056	-
		1.50	.049	-			.50	.069	-
1.44	.125	2.00	.049	-	2.43	.078	.75	.068	-
		0.25	.056	-			1.00	.071	-
		.50	.069	-			1.25	.069	-
		.75	.068	-			1.50	.067	-
		1.00	.071	-			2.00	.067	-
1.44	.125	1.25	.070	-	2.43	.078	.25	.053	-
		1.50	.071	-			.50	.061	-
							.75	.067	-

Table 8 (Con't)

$\lambda$	$\tau$	$b$	$\eta_2 = 0.12$ $\eta_{s1} \quad \eta_{s2}$		$\lambda$	$\tau$	$b$	$\eta_2 = 0.20$ $\eta_{s1} \quad \eta_{s2}$	
2.88	.078	0.125	.015	-	0.123	.078	0.25	-	.040
		.25	.073	-			.50	1.01	.095
		.50	.103	-			.75	1.01	.100
		.75	.102	-			1.00	.670	.225
		1.00	.118	-			1.25	.760	.170
		1.25	.114	-			1.50	.470	.380
		1.50	.114	-			2.00	.360	.765
		2.00	.112	-	0.227	.078	0.25	.685	.095
3.26	.078	0.25	.085	-			.50	.505	.210
		.50	.106	-			.75	.470	.260
		.75	.114	-			1.00	.330	.610
		1.00	.120	-			1.25	.360	.450
		1.25	.114	-			1.50	.240	-
		1.50	.112	-			2.00	.210	-
3.42	.078	0.125	.080	-	0.441	.078	0.125	.270	.185
		.25	.102	-			.25	.240	.240
		.50	.102	-			.50	.175	.250
		.75	.107	-	0.446	.078	0.125	.270	.148
		1.00	.112	-			.25	.294	.176
		1.25	.110	-			.50	.180	-
		1.50	.102	-	0.567	.032	0.50	.170	.057
		2.00	.096	-			.75	.160	.140
3.75	.078	0.25	.102	-			1.00	.132	.185
		.50	.128	-			1.25	.120	.230
		.75	.130	-			1.50	.110	.240
		1.00	.141	-	0.642	.032	0.25	-	.040
		1.25	.136	-			.50	-	.090
		1.50	.137	-			.75	.168	.156
3.82	.078	0.125	.114	-			1.00	.145	.270
		.25	.138	-			1.25	.140	-
		.50	.142	-			1.50	.134	-
		.75	.146	-	0.785	.125	0.25	.114	-
		1.00	.144	-			.50	.118	-
		1.25	.155	-			.75	.120	-
		1.50	.138	-			1.00	.122	-
		2.00	.132	-			1.25	.124	-
4.42	.078	0.125	.166	-			1.50	.124	-
		.25	.240	-	0.986	.125	0.25	.078	-
		.50	.269	-			.50	.080	-
		.75	.231	-			.75	.079	-
		1.00	.202	-			1.00	.077	-

Table 8. (Con't)

$\lambda$	$\tau$	$b$	$\eta_2 = 0.20$		$\lambda$	$\tau$	$b$	$\eta_2 = 0.20$	
			$\eta_{s1}$	$\eta_{s2}$				$\eta_{s1}$	$\eta_{s2}$
1.04	.125	0.25	.078	-	2.37	.035	0.25	.127	-
		.50	.081	-			.50	.138	-
		.75	.083	-			.75	.141	-
		1.00	.080	-			1.00	.142	-
		1.25	.080	-			1.25	.138	-
							2.00	.144	-
1.63	.125	0.25	.120	-	3.25	.032	0.25	.051	-
		.50	.122	-		.062		.147	-
		.75	.123	-		.078		.136	-
		1.00	.123	-		.125		.122	-
		1.25	.123	-					
1.67	.125	0.25	.114	-	3.84	.078	0.125	.170	-
		.50	.127	-			.25	.196	-
		.75	.129	-			.50	.200	-
		1.00	.128	-			.75	.202	-
		1.25	.127	-			1.00	.200	-
		1.50	.130	-					

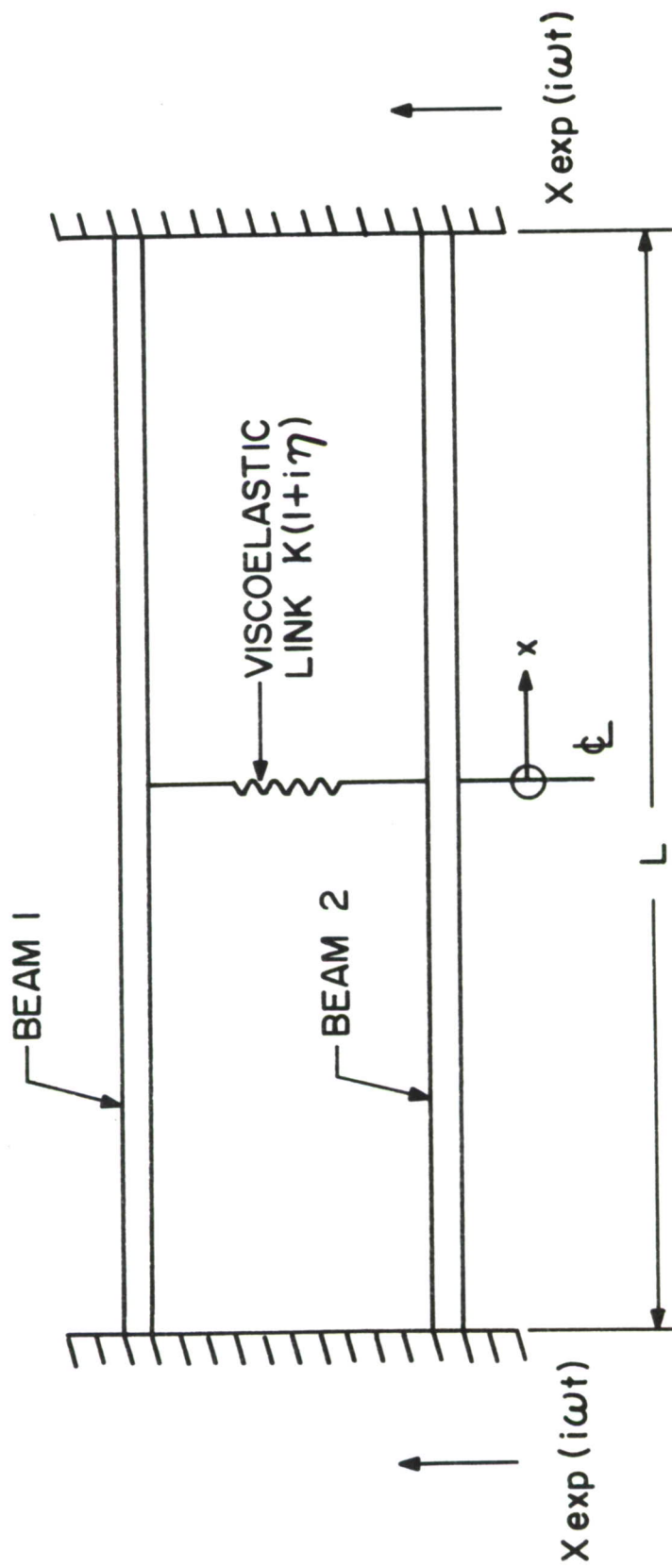


Figure 1. Idealized Link Joining Parallel Beams

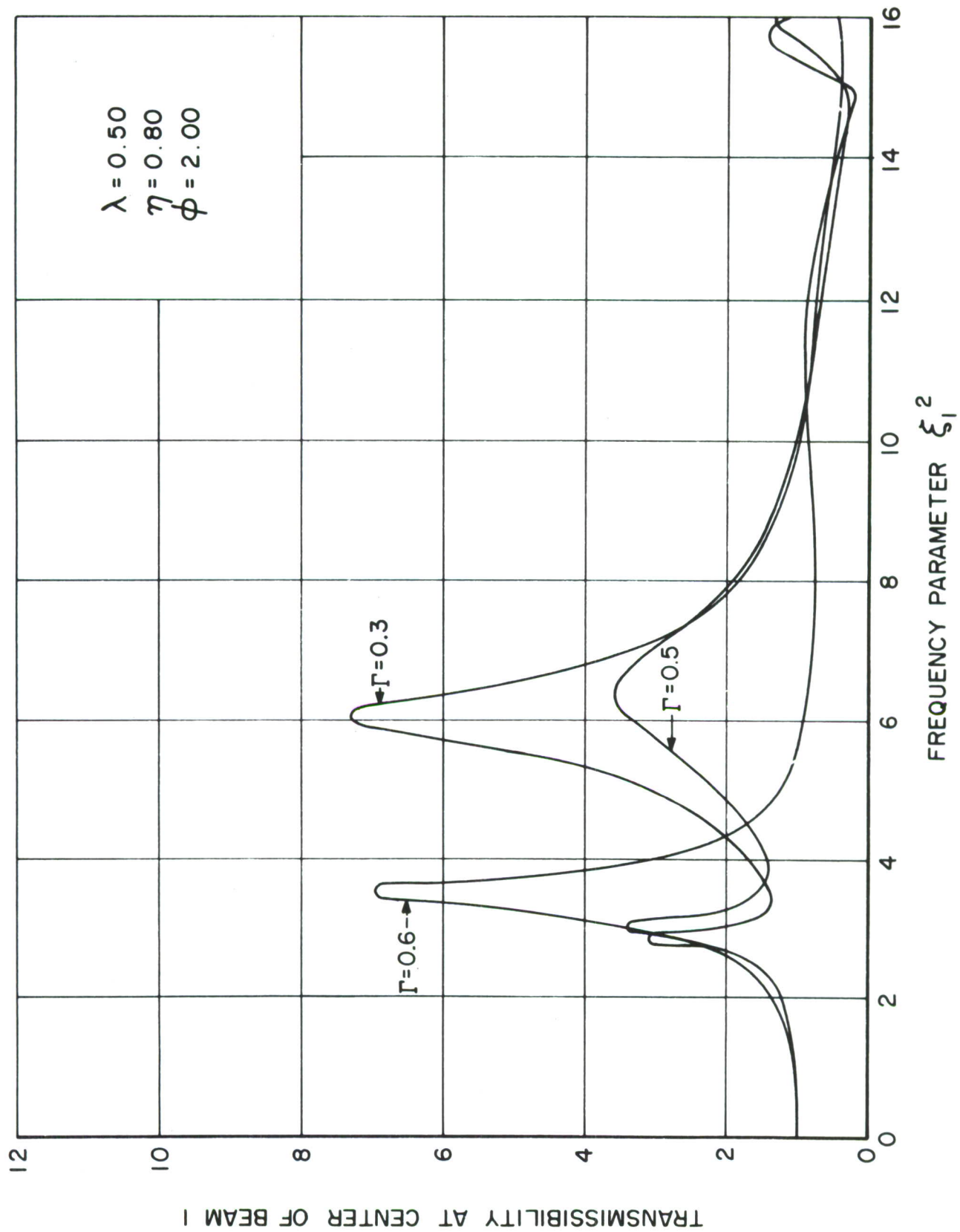


Figure 2. Typical Transmissibility Spectra for  $\lambda/\phi < 1$

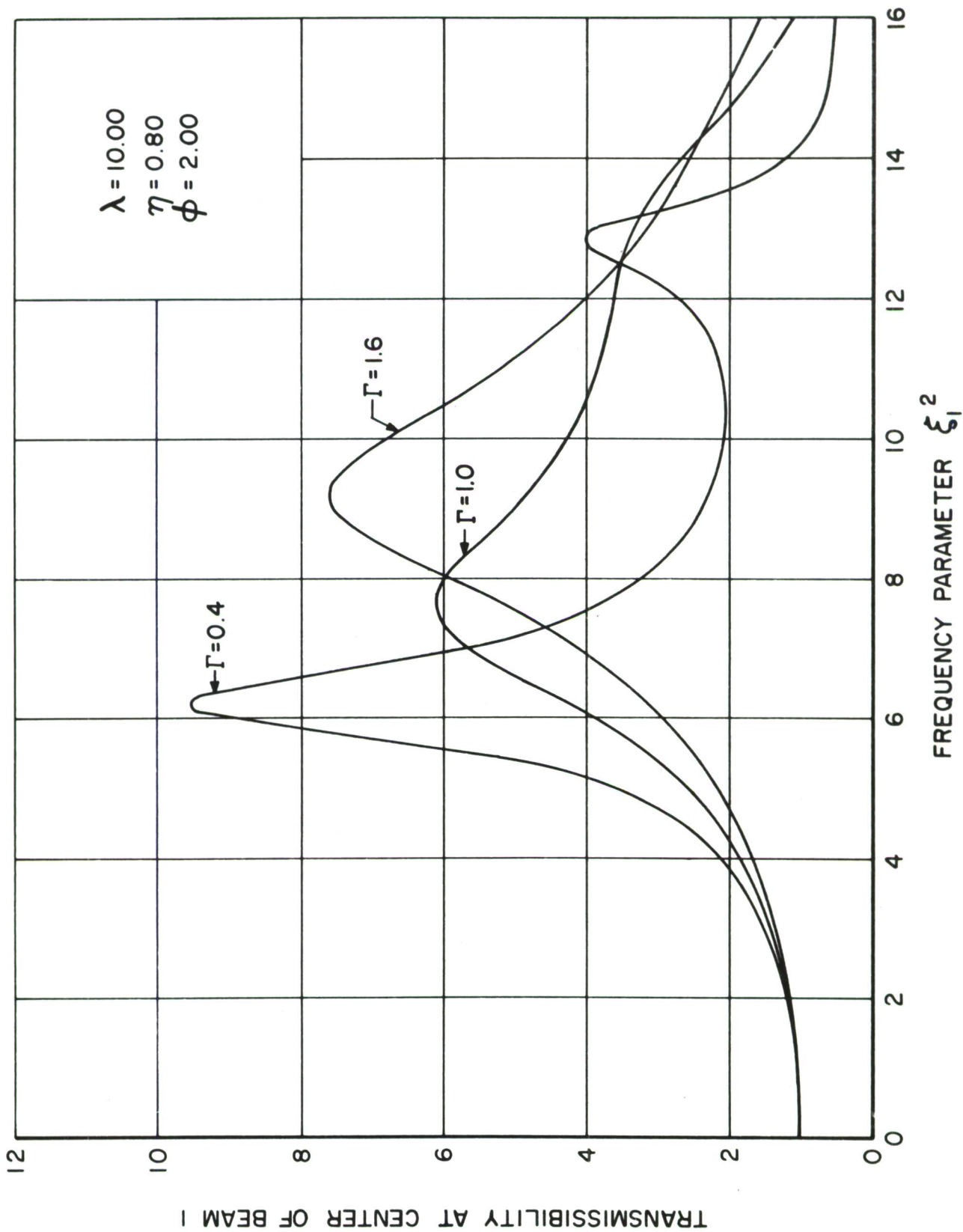


Figure 3. Typical Transmissibility Spectra for  $\lambda/\phi > 1$

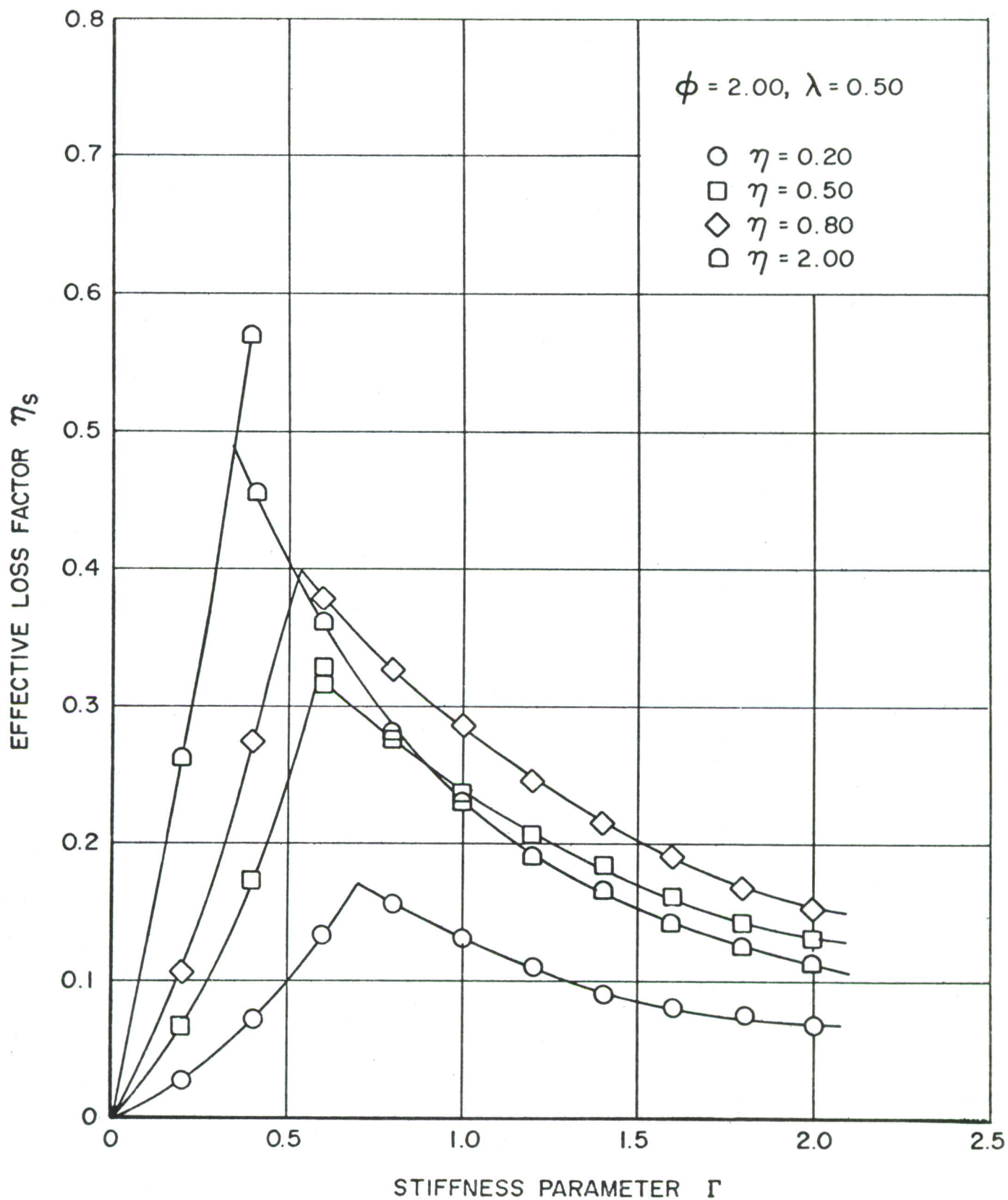


Figure 4. Typical Graphs of Effective Loss Factor Against Link Stiffness Parameter for  $\lambda/\phi < 1$

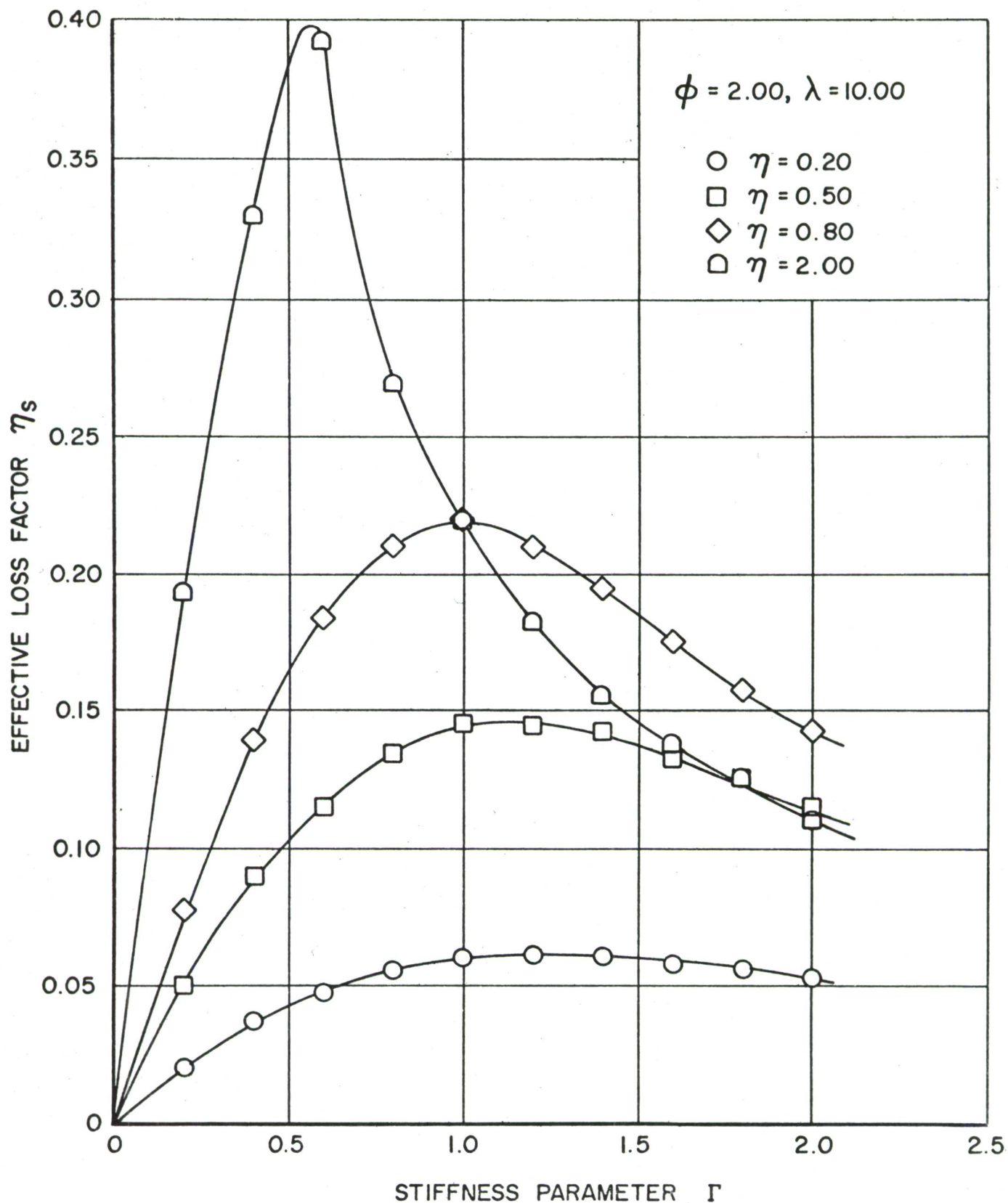


Figure 5. Typical Graphs of Effective Loss Factor Against Link Stiffness Parameter for  $\lambda/\phi > 1$

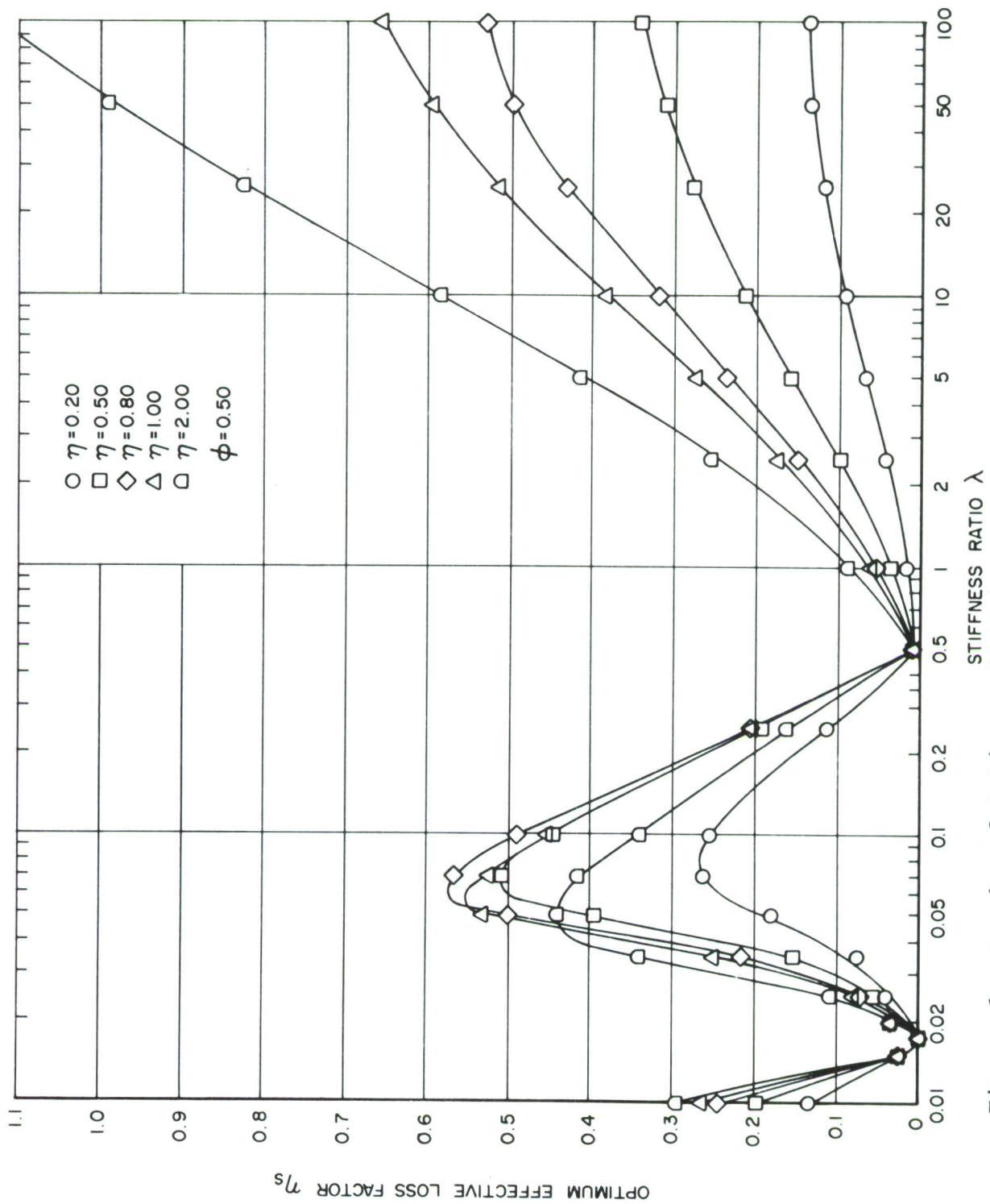


Figure 6. Graphs of Optimum Effective Loss Factor Against Beam Stiffness  
Parameter for  $\phi = 0.50$  and  $\eta_2 = 0$

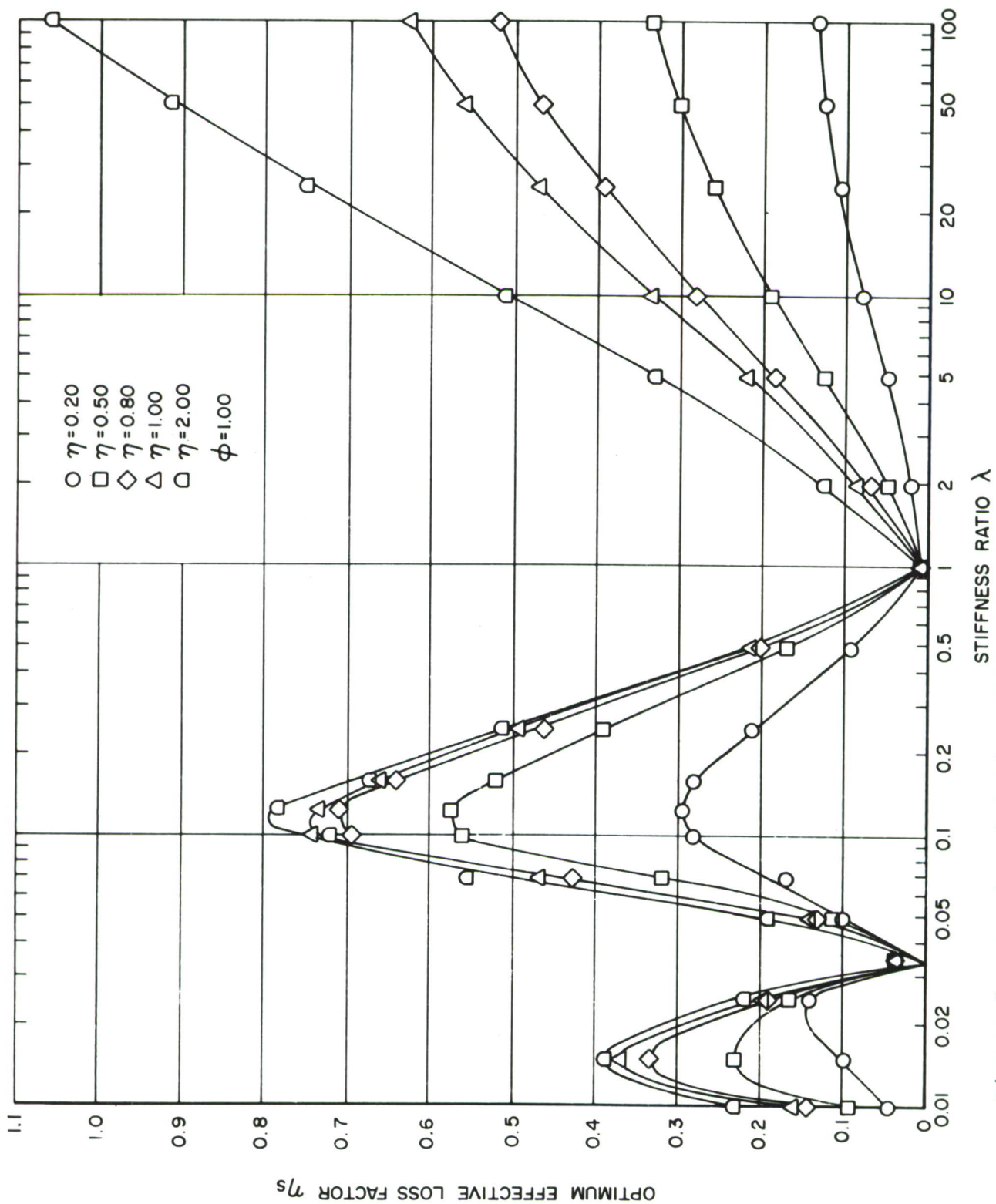


Figure 7. Graphs of Optimum Effective Loss Factor Against Beam Stiffness  
Parameter for  $\phi = 1.0$  and  $\eta_2 = 0$

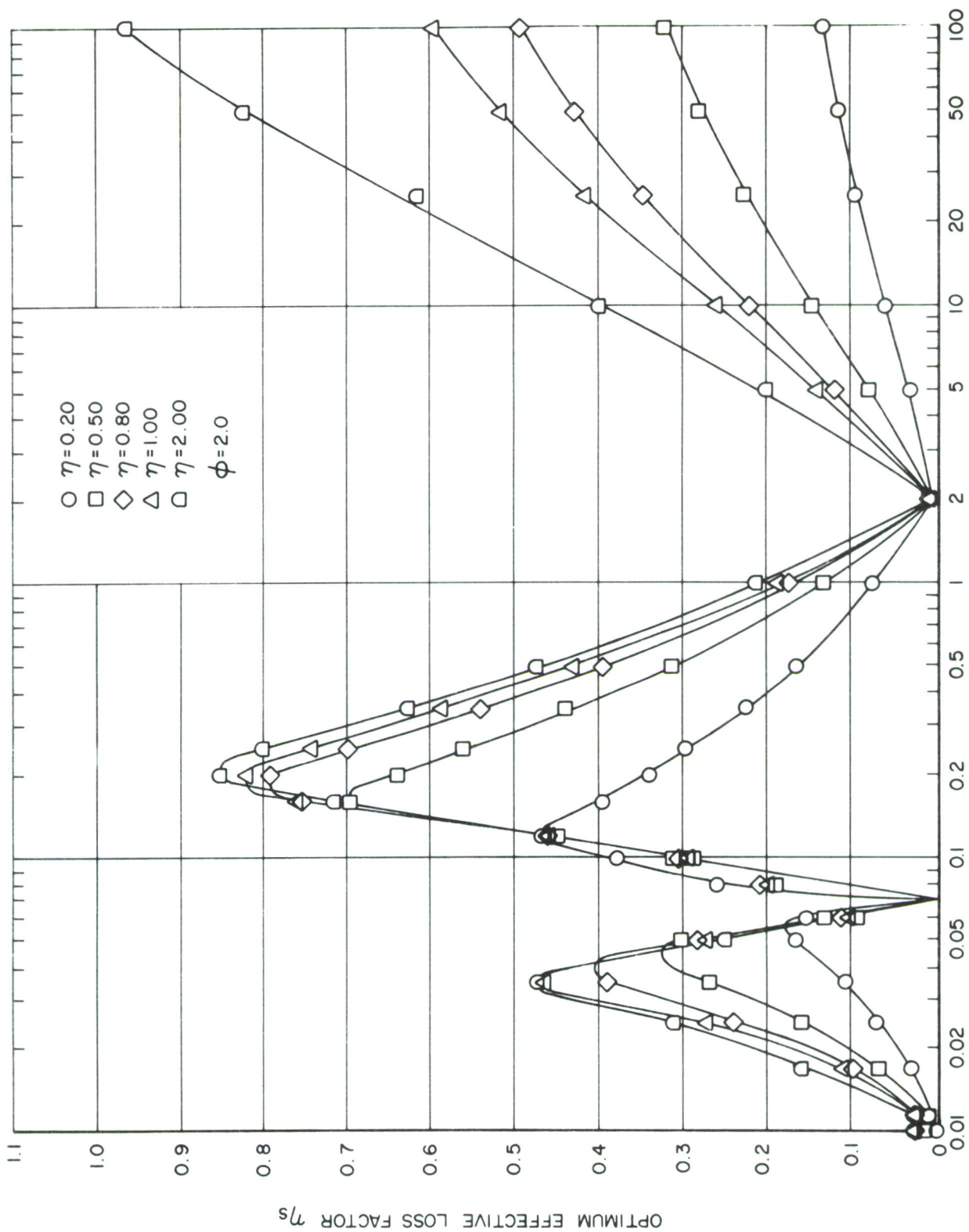


Figure 8. Graphs of Optimum Effective Loss Factor Against Beam Stiffness  
Parameter for  $\phi = 2.0$  and  $\eta_2 = 0$

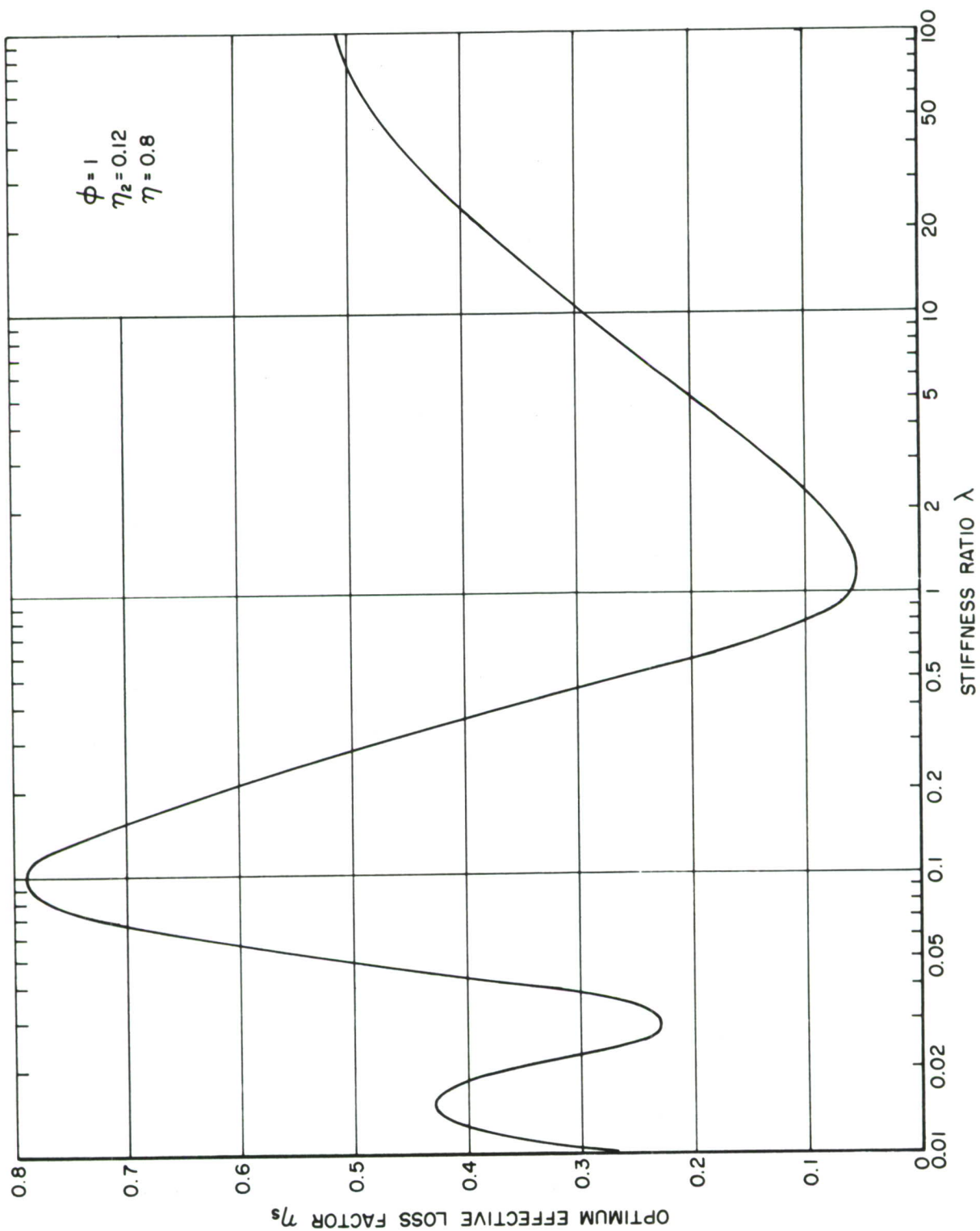


Figure 9. Graph of Optimum Effective Loss Factor Against Beam Stiffness Parameter for  $\phi = 1.0$  and  $\eta_2 = 0.12$

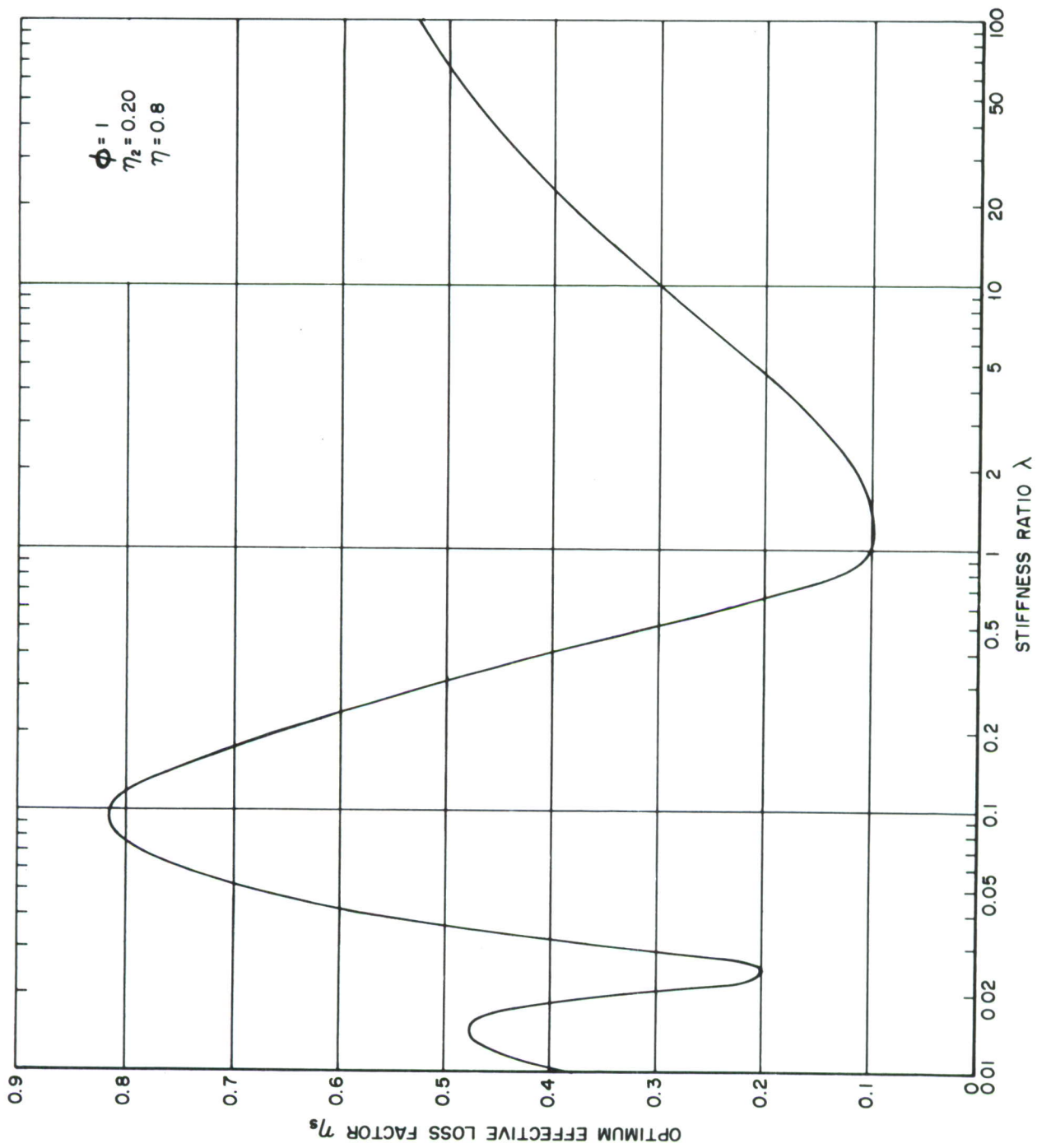


Figure 10. Graph of Optimum Effective Loss Factor Against Beam Stiffness Parameter  
for  $\phi = 1.0$  and  $\eta_2 = 0.20$

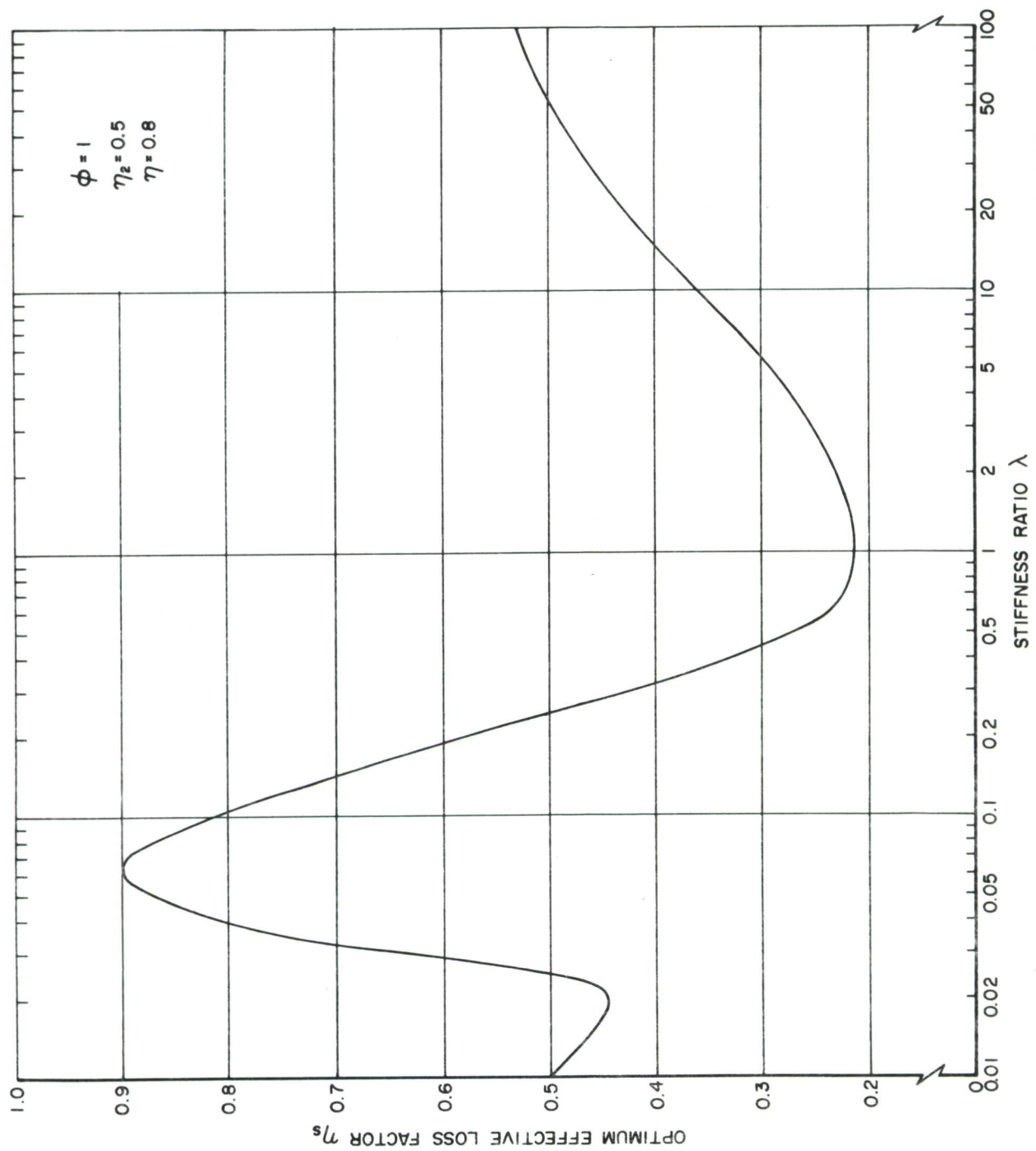


Figure 11. Graph of Optimum Effective Loss Factor Against Beam Stiffness Parameter  
for  $\phi = 1.0$  and  $\eta_2 = 0.50$

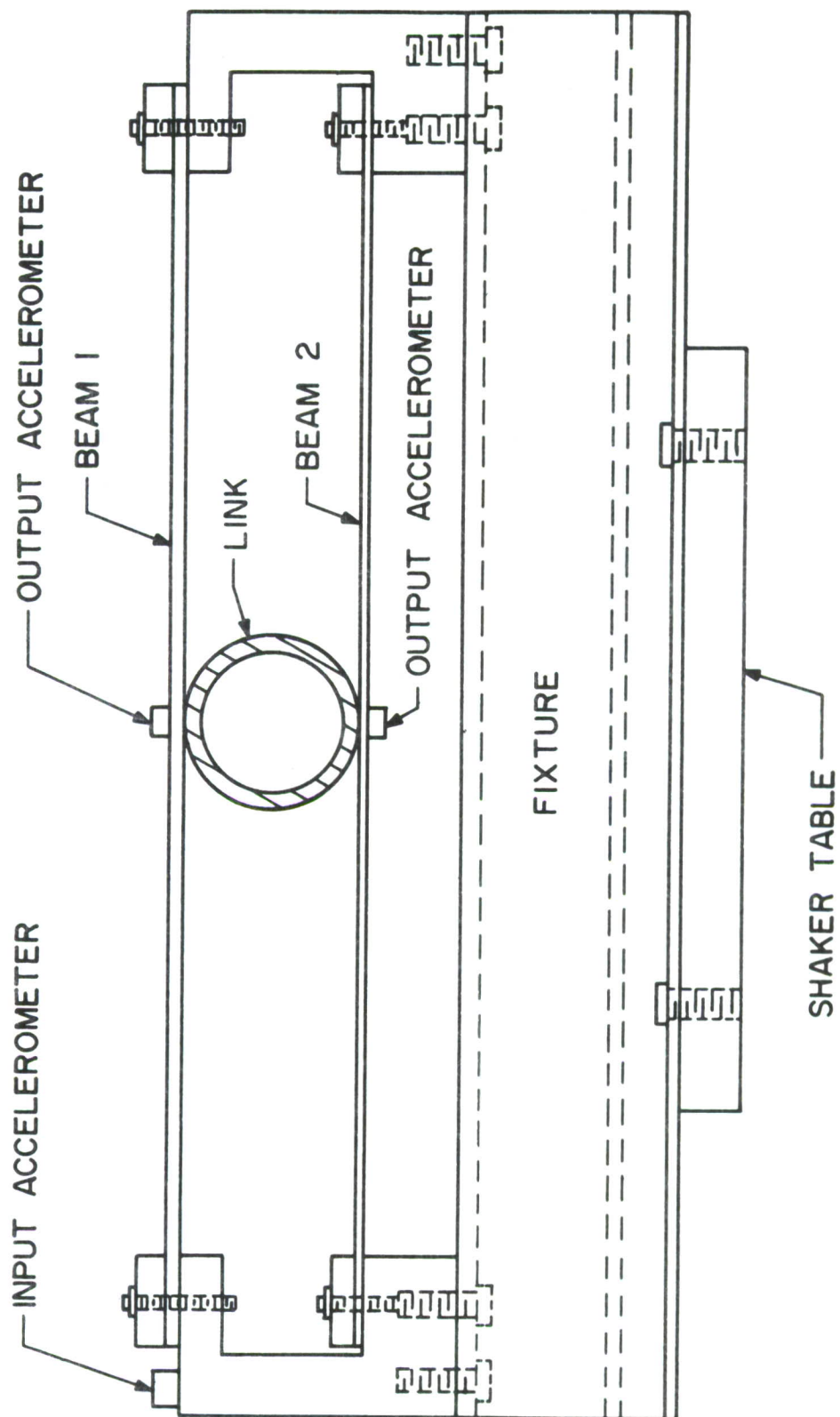


Figure 12. Sketch of Apparatus

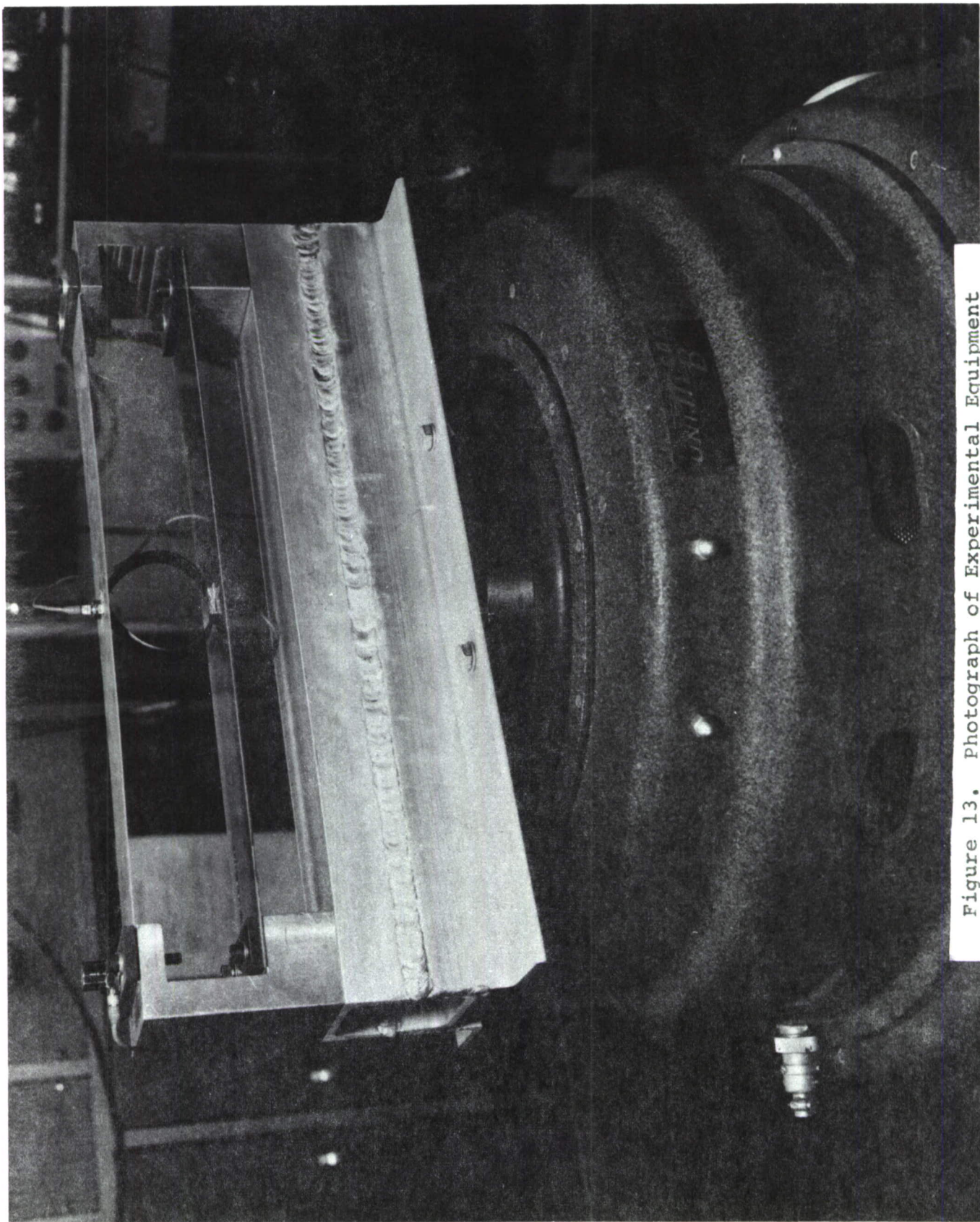


Figure 13. Photograph of Experimental Equipment

$\lambda = 0.50$ ,  $\eta = 0.8$ ,  $\phi = 2.0$   
 INPUT ACCELERATION = 1.0 g  
 LINK THICKNESS  $\tau = 0.078$  IN.  
 BEAM LENGTH  $L = 16.0$  IN  
 LINK DIAMETER  $d = 2.5$  IN.

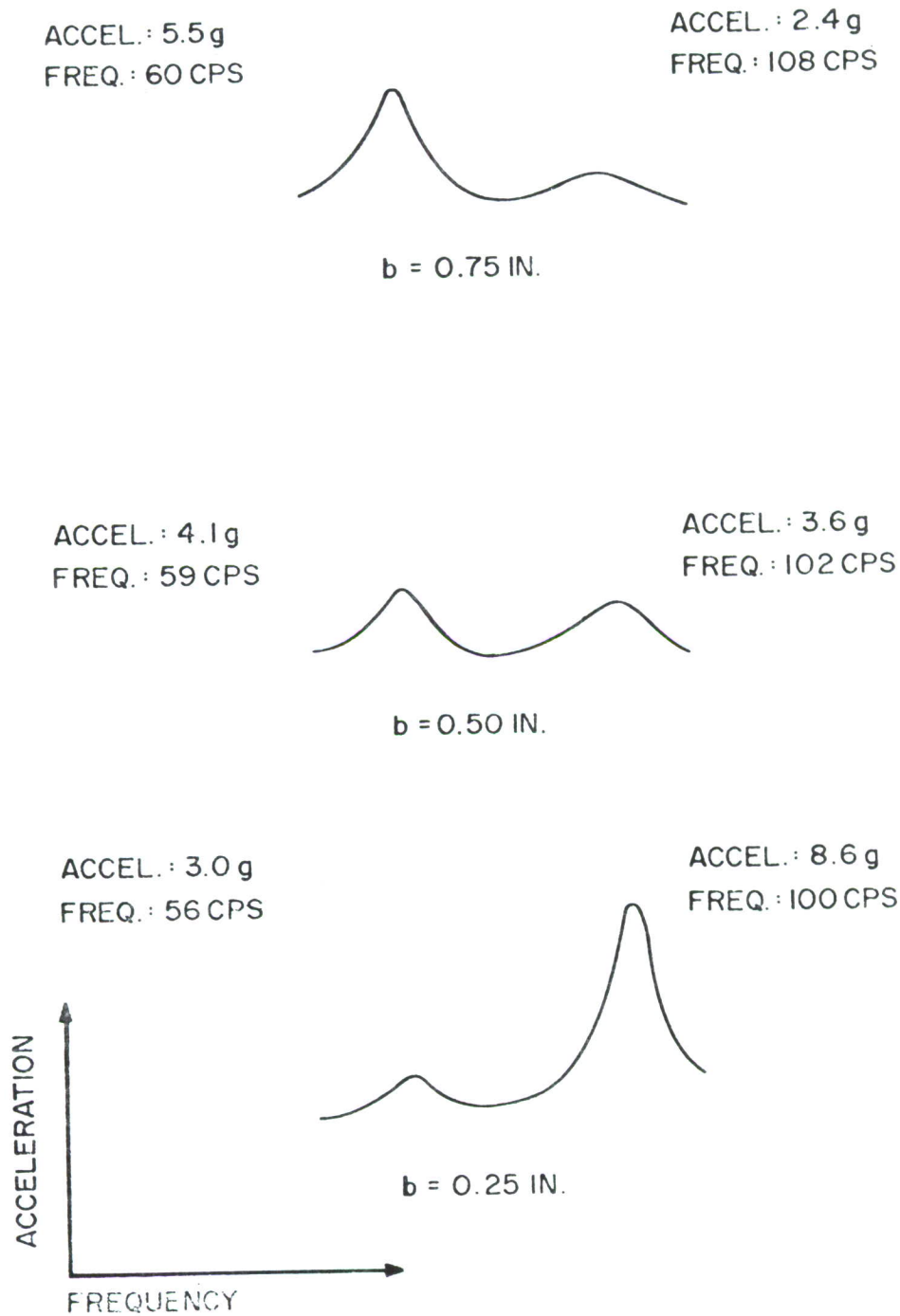
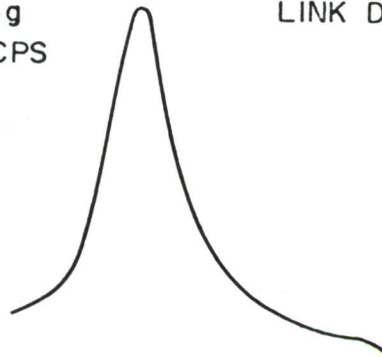


Figure 14. Typical Measured Response Spectra for  $\lambda/\phi < 1$

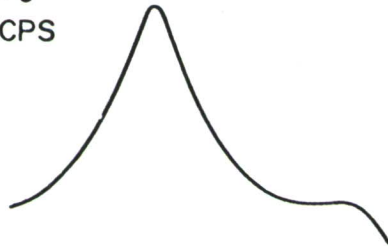
$\lambda = 8.0, \eta = 0.8, \phi = 2.0$   
INPUT ACCELERATION = 1.0g  
LINK THICKNESS  $\tau = 0.078$  IN.  
BEAM LENGTH  $L = 16.0$  IN.  
LINK DIAMETER  $d = 2.5$  IN.

ACCEL: 11.3g  
FREQ: 74 CPS



$b = 1.00$  IN.

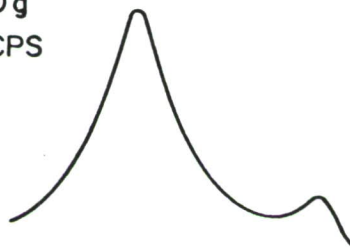
ACCEL: 8.5g  
FREQ: 65 CPS



ACCEL: 2.6g  
FREQ: 105 CPS

$b = 0.50$  IN.

ACCEL: 9.0g  
FREQ: 64 CPS



ACCEL: 3.3g  
FREQ: 107 CPS

$b = 0.25$  IN.

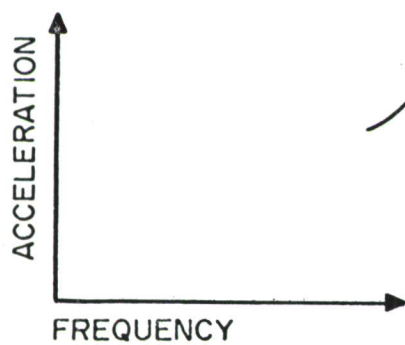


Figure 15. Typical Measured Response Spectra for  $\lambda/\phi > 1$

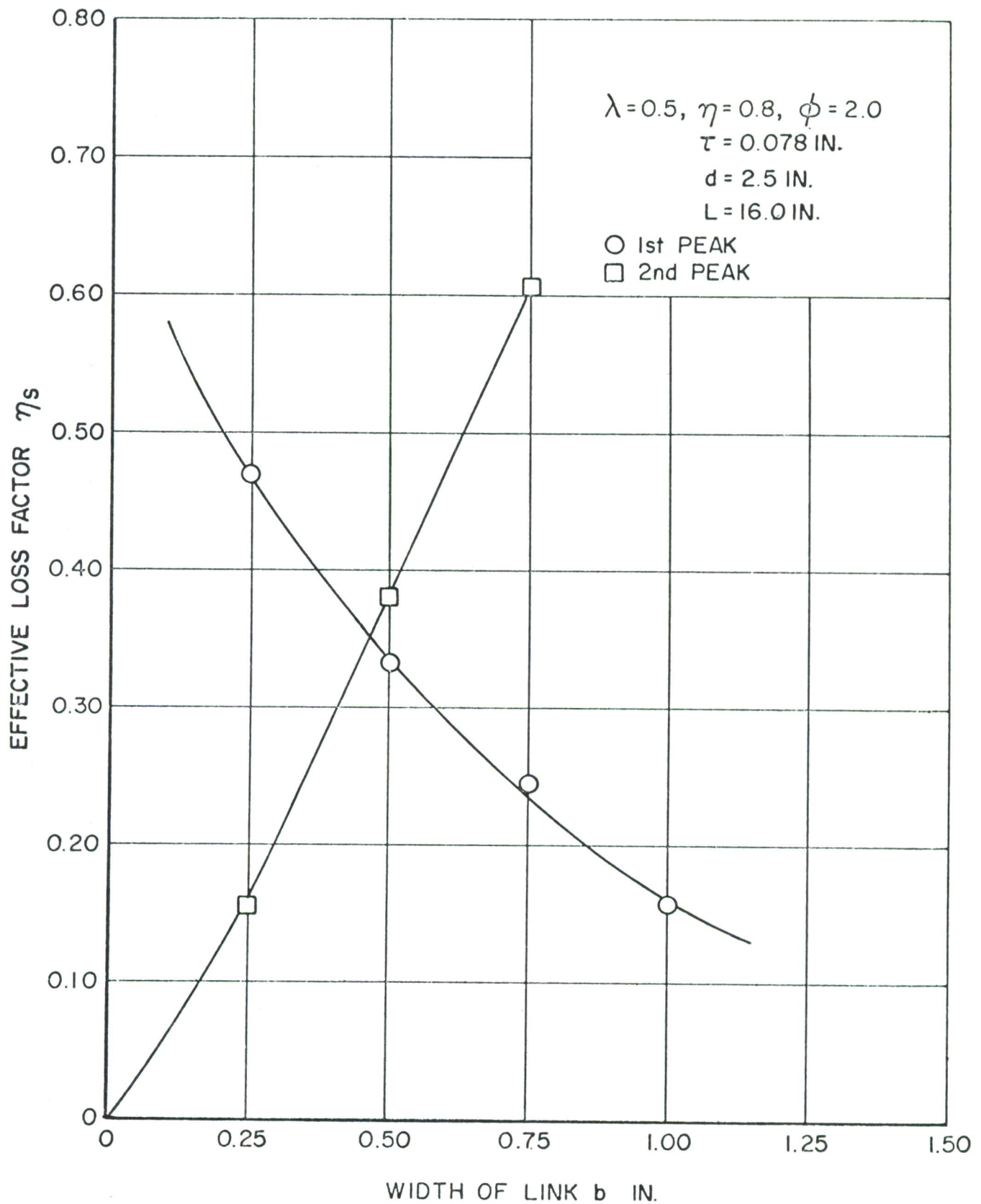


Figure 16. Graphs of Experimentally Measured Values of Effective Loss Factor Against Link Width for  $\lambda/\phi < 1$

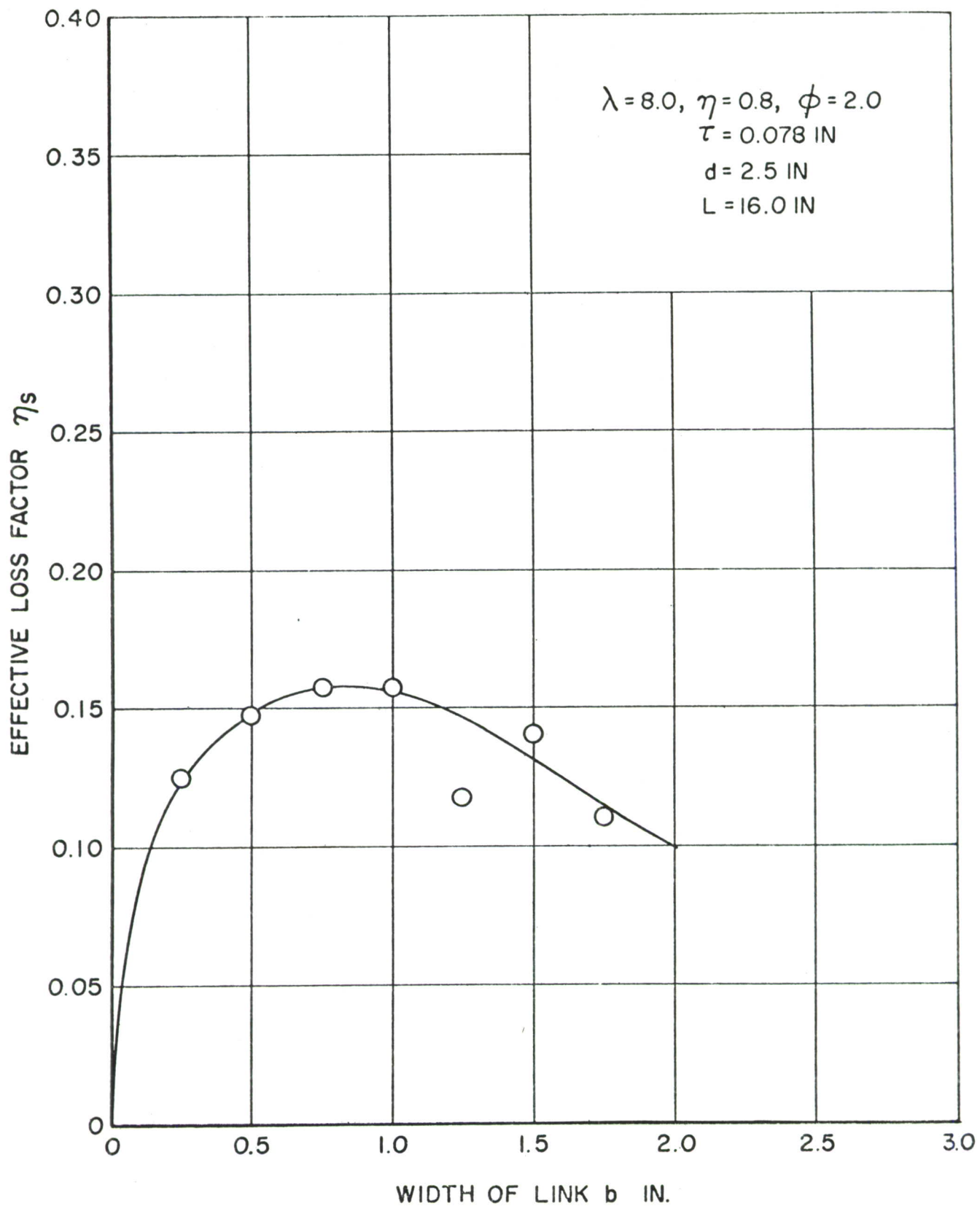


Figure 17. Graphs of Experimentally Measured Values of Effective Loss Factor Against Link Width for  $\lambda/\phi > 1$

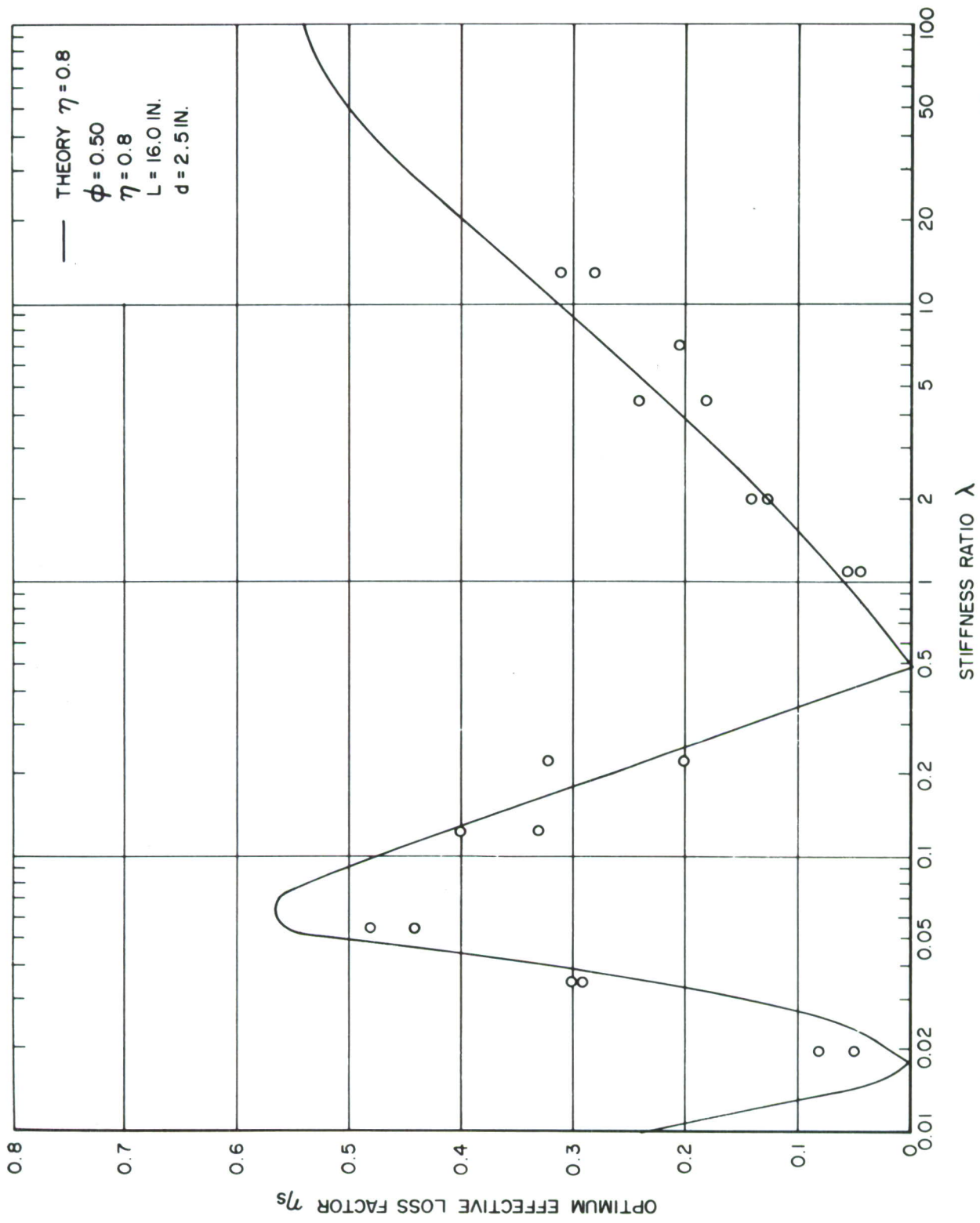


Figure 18. Comparison of Experimental and Theoretical Values of Optimum Effective Loss Factors for  $\phi = 0.5$  and  $\eta_2 = 0$

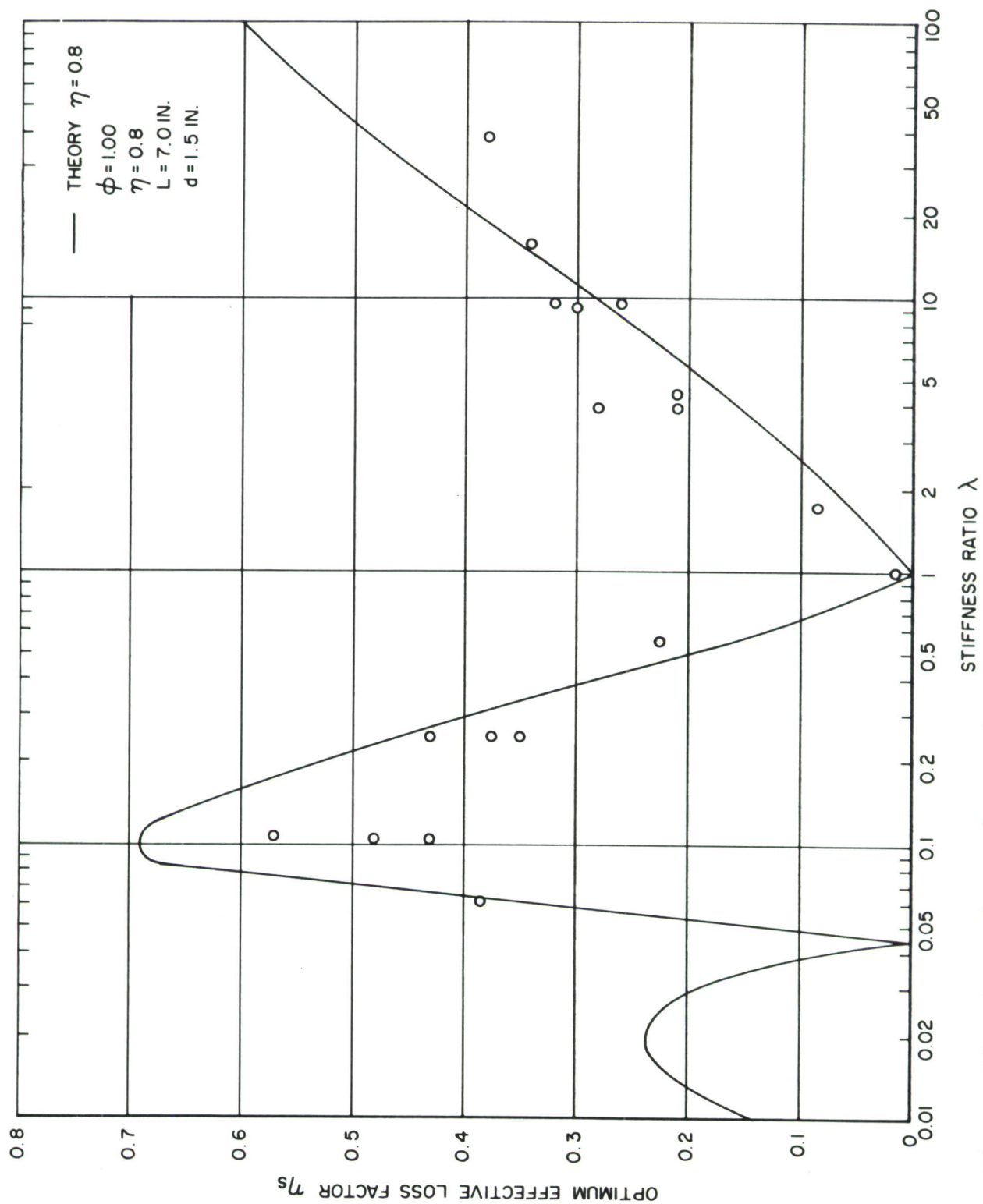


Figure 19. Comparison of Experimental and Theoretical Values of Optimum Effective  
 Loss Factors for  $\phi = 1.0$  and  $\eta_2 = 0$

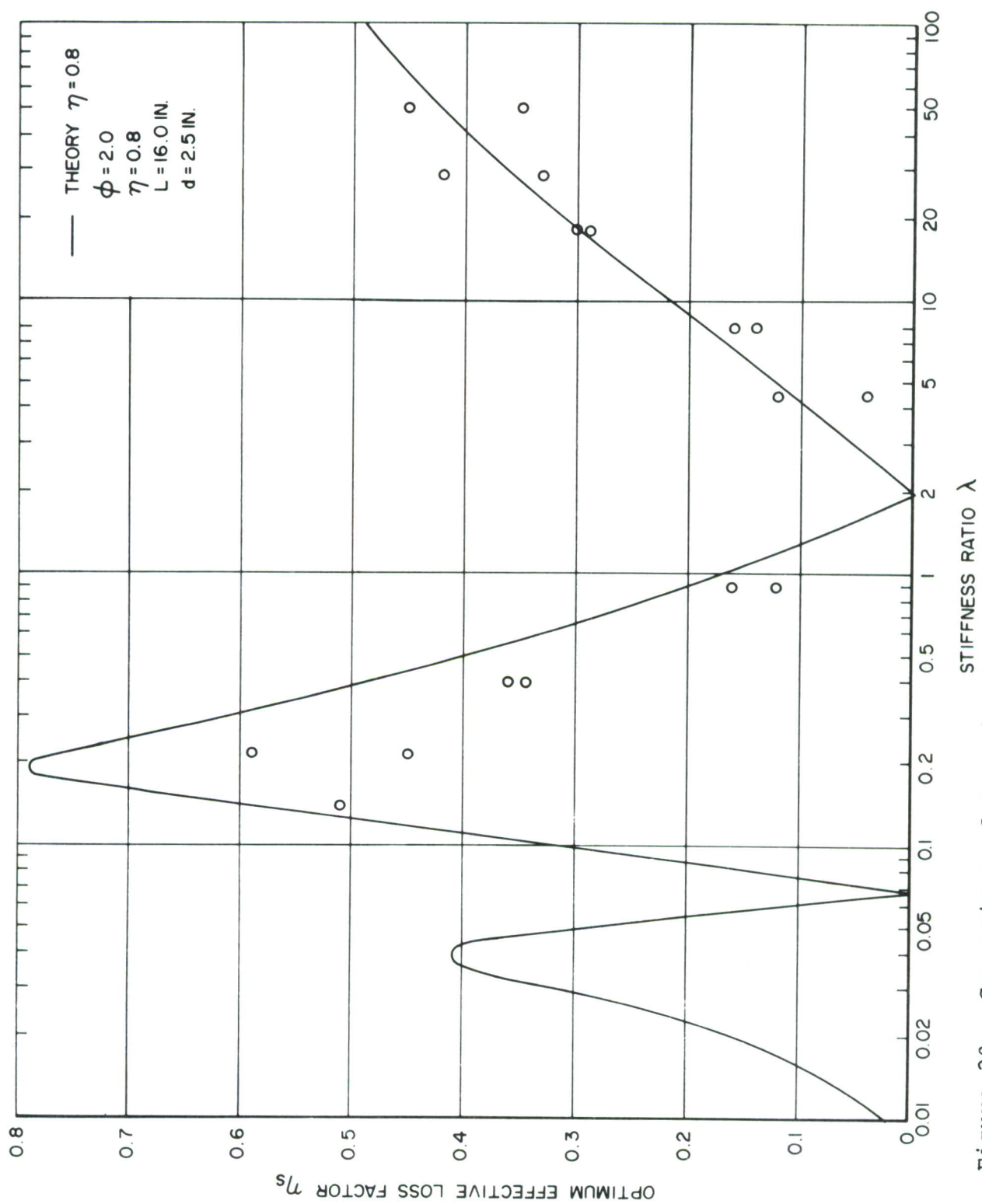


Figure 20. Comparison of Experimental and Theoretical values of Optimum Effective Loss Factors for  $\phi = 2.0$  and  $\eta_2 = 0$

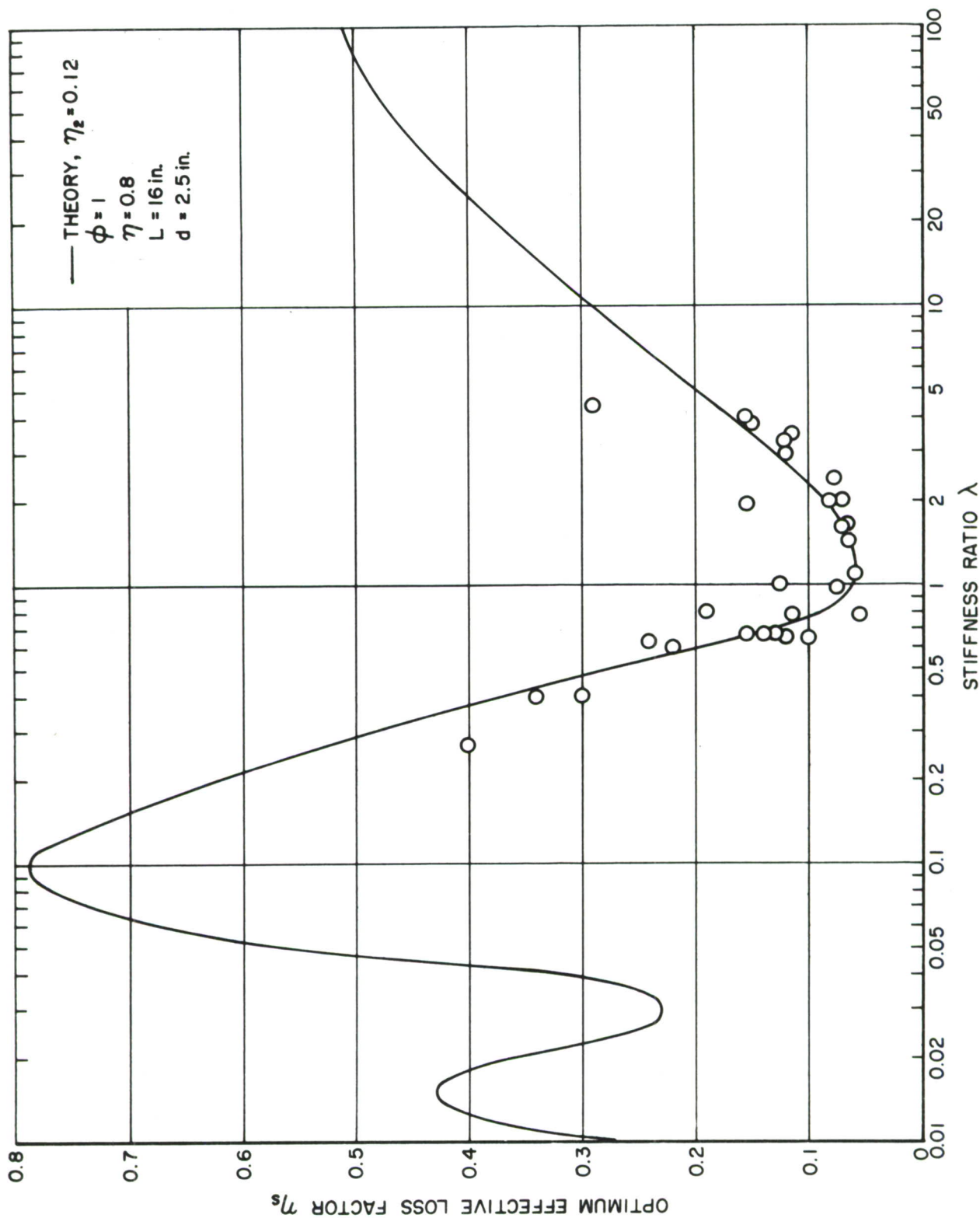


Figure 21. Comparison of Experimental and Theoretical Values of Optimum Effective Loss Factors for  $\phi = 1.0$  and  $\eta_2 = 0.12$

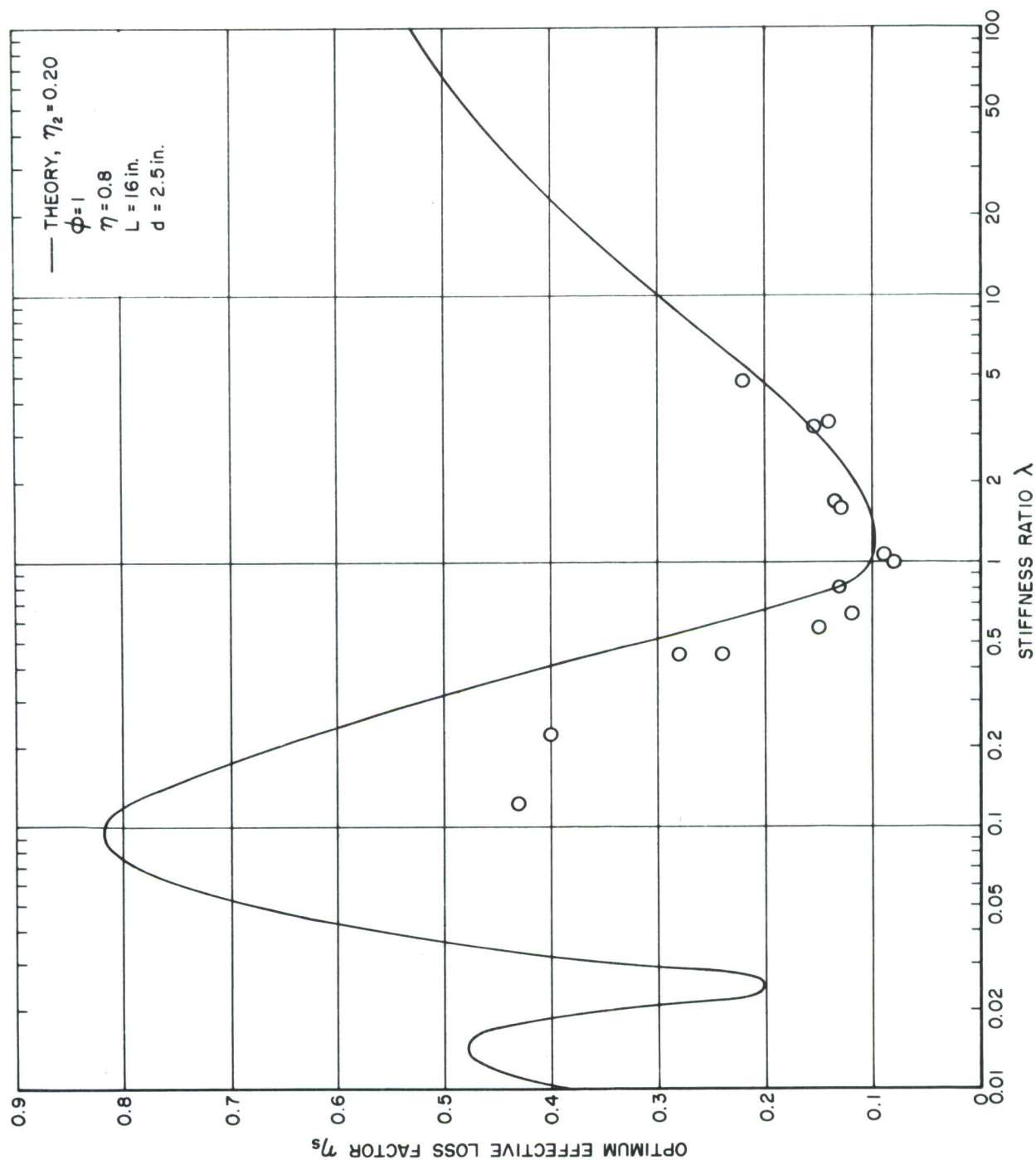


Figure 22. Comparison of Experimental and Theoretical Values of Optimum Effective Loss Factors for  $\phi = 1.0$  and  $\eta_2 = 0.20$

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**11. SUPPLEMENTARY NOTES**

**12. SPONSORING MILITARY ACTIVITY**

Metals and Ceramics Division  
Air Force Materials Laboratory  
Wright-Patterson AFB, Ohio

**13. ABSTRACT**

In this report, an analysis is presented for the response and effective damping of two parallel clamped-clamped beams with a single viscoelastic link joining their centers and with various amounts of internal damping in one beam. It is shown that high damping can still be achieved, even when the fundamental frequency of the beam in question is equal to any natural frequency of the other beam.

The experimental investigation of the same configuration is described and it is shown that the theory accurately describes the phenomena occurring for one particular value of the loss factor of the link, which comprised a ring of one commercially available viscoelastic material.

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